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National Center and State Collaborative

## NCSC Math Activities with Scripted Systematic Instruction (MASSI): Middle School Ratio and Proportion

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National Center and State Collaborative
The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for postsecondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

The contents of this document were developed as part of the National Center and State Collaborative by Julie Thompson, Alicia Saunders, and Diane Browder at University of North Carolina at Charlotte and verified by Amy Lehew, math content expert, under a grant from the Department of Education (PR/Award \#: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the U.S. Department of Education and no assumption of endorsement by the Federal government should be made.

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National Center and State Collaborative

# NCSC Math Activities with Scripted Systematic Instruction (MASSI): Middle School Ratio and Proportion 

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## MASSI: Math Activities with Scripted Systematic Instruction

## Activity: Store Rewards Programs

## Grade Band: Grades 6-8

## Concept: Ratio and Proportion

| Common Core State Standard | Core Content Connectors | MASSI OBJECTIVES |
| :---: | :---: | :---: |
| 6.RP. 1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." | $6^{\text {th }}$ 6.PRF.1c1 Describe the ratio relationship between two quantities for a given situation. | Students will write a ratio that matches a pictured ratio relationship. |
| 7.RP. 2 Recognize and represent proportional relationships between quantities. <br> - Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. <br> - Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. <br> - Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t=p n$. <br> - Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where $r$ is the unit rate. | 7.NO. $2 f 1$ Identify the proportional relationship between two quantities. | Given a chart, students will calculate the proportional relationship between two items. |
| 8.F. 5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. | $8^{\text {th }}$ <br> 8.NO.3c3 Analyze provided information (e.g., a graph) to describe the relationship between two quantities. | 1. Given a graph, students will determine how much of an item is needed to gain 1 of another item (e.g., how many paid games of putt-putt must you play to get 1 free game?). <br> 2. Using manipulatives, demonstrate how many items are needed to obtain another item in a two-step problem. |

Be sure to provide specific practice to students on the skills that correspond to their grade level.

Teaching Materials: Movie ratio matching cards, cinema ticket and popcorn counters, frequent buyer cards, ratio school supply flyer, proportional relationship charts, face pictures with matching ratio cards, proportional relationship graphic organizer, proportional relationship graphs with conversion charts, ticket counters, stop sign, better buck graphic organizer, altogether mat.

Other Materials: calculators
Worksheets: There are student worksheets to review each component of the lesson.
Assessments: Progress Monitoring for taking data during the lesson; Skills Test
TEACHING OVERVIEW: The first section of the MASSI provides practice on identifying a ratio, demonstrating understanding of the constancy of ratios, and determining sets within a ratio that are more. $6^{\text {th }}$ graders will learn to write a ratio that matches a pictured ratio relationship. This is a good review for your $7^{\text {th }}$ and $8^{\text {th }}$ graders. The $7^{\text {th }}$ graders will be calculating the proportional relationship between two items. This is useful for your $6^{\text {th }}$ and $8^{\text {th }}$ graders to gain fluency with these concepts and practicing numeracy and operation skills. Then $8^{\text {th }}$ graders work on analyzing a graph to determine how much of an item is needed to obtain 1 of another item as well as using manipulatives to demonstrate how many items are needed to obtain another item when solving a two-step problem.

## SCRIPT FOR LESSON

## BUILDING ESSENTIAL UNDERST ANDING: Given picture or graph of two sets, select the set that has more. (Skip this section for students who understand these relationships and can identify these concepts).

INTRODUCE PROBLEM: "Did you know that some stores give away free items for buying a certain number of things? These stores use ratios to tell us what we need to do to earn the free item. A ratio compares two amounts. Today we will learn to solve problems using ratios."

- $\quad$ - MODEL THE PROCESS: "First, let's look at some ratios and practice identifying which set has more." Display cinema ticket counters. Separate them into two groups with one clearly more than the other (e.g., 18 in one group and 2 in the other). Point to the larger group and say, "This is more." Shuffle the counters and make two more groups. Point to the larger group and say, "This is more." Continue demonstrating examples - "This is more." And non-examples "This is not more" Use the EXAMPLE/NON-EXAMPLE script as a guide to teach the concept of more.
 STUDENT PRACTICE: Now it's your turn. Use CONSTANT TIME DELAY script as needed to help students.
**Note: As you display the counters be sure to randomly vary the position of your answers to ensure that students aren't just memorizing the position of the answer. (e.g., answers are located: left, left, right, top, right, bottom, left, top, right, right, bottom, etc.)
CHECK AND SCORE

| Step | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{1 .}$ | Separate counters into two groups with one more than the other. <br> Show me more. | Student points to or indicates the set that has more. |
| $\mathbf{2 .}$ | Separate counters into two groups with one more than the other. <br> Show me more. | Student points to or indicates the set that has more. |
| $\mathbf{3 .}$ | Separate counters into two groups with one more than the other. <br> Show me more. | Student points to or indicates the set that has more. |
| $\mathbf{4 .}$ | Separate counters into two groups with one more than the other. <br> Show me more. | Student points to or indicates the set that has more. |
| $\mathbf{5 .}$ | Separate counters into two groups with one more than the other. <br> Show me more. | Student points to or indicates the set that has more. |

- MODEL THE PROCESS: "Often ratios are made up of two different types of sets. Let's practice identifying more with movie ticke ts and popcorn this time." Display cinema ticket and popcorn counters. Separate them into two groups with one clearly more than the other (e.g., 18 in one group and 2 in the other). Point to the larger group and say, "This is more." Shuffle the counters and make two more groups. Point to the larger group and say, "This is more." Continue demonstrating examples - "This is more." And non-examples "This is not more" Use the EXAMPLE/NON-EXAMPLE script as a guide to teach the concept of more.

STUDENT PRACTICE: Now it's your turn. Use CONSTANT TIME DELAY script as needed to help students.
**Note: For student practice, use the teacher material cards that look like the one below.


CHECK AND SCORE

| Step | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{1 .}$ | Display the 2:4 ticket/popcorn card. Show me more. | Student points to or indicates the popcorn set. |
| $\mathbf{2 .}$ | Display the 4:2 ticket/popcorn card. Show me more. | Student points to or indicates the ticket set. |

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| 3. | Display the 6:3 ticket/popcorn card. Show me more. | Student points to or indicates the ticket set. |
| :---: | :--- | :--- |
| $\mathbf{4 .}$ | Display the 4:1 ticket/popcorn card. Show me more. | Student points to or indicates the ticket set. |
| 5. | Display the 2:5 ticket/popcorn card. Show me more. | Student points to or indicates the popcorn set. |

## BUILDING ESSENTIAL UNDERST ANDING: CONCEPT AND SYMBOLS: Select a simple ratio that matches a ratio relationship. (Skip this section for students who understand these relationships and can identify these concepts).

INTRODUCE PROBLEM: "Now let's practice matching two groups of items to a ratio. Remember some places offer rewards for buying things." Display 6 cinema tickets and 2 popcorn pictures. Here are some movie tickets and some popcorn pictures. At the Prescott Theater if you buy some tickets you get some free popcorn. I need to find the ratio of popcorn to movie tickets.

MODEL THE PROCESS: Display answer choices. Here are my answer choices. Before I choose my answer, I need to count the tickets and popcorn. First I will count the tickets. Point to the tickets as you count. Count with me. One. Two. Three. Four. Five. Six. There are six movie tickets. Now I will count the popcorn. Point to the popcorn as you count. Count with me. One. Two. There are two popcorns. Point to answer choices. Now I will choose my answer. I know that there are 6 movie tickets and two popcorns. Point to corresponding answers as you read. This says: 2 tickets to 4 popcorns. This says: 5 tickets to 5 popcorns. And, this says: 6 tickets to 2 popcorns. Point to correct answer as you say, 6 tickets to 2 popcorns is the answer. At Prescott Theater if I buy 6 movie tickets I will get two free popcorns.

( $)$STUDENT PRACTICE: Now it's your turn to practice two problems. Use the CONSTANT TIME DELAY script to teach students to identify the pictured ratio.
${ }^{* *}$ Note: After students master solving these two problems, to ensure the concept is understood vary the ratios each day. 20 pictures each of popcorn and movie tickets have been provided. You can laminate them, place Velcro on the back and create a problem and answer choice boards with Velcro and a place to write the ratios. Also consider using other pictures/scenarios to ensure generalization.

CHECK AND SCORE

| Step | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{1 .}$ | Display Wood Theater problem. At Wood Theater if you buy <br> some tickets you get some free popcorn. Find the ratio of <br> popcorn to movie tickets. Display answer choices. | Selects ratio 4:1 |
| $\mathbf{2 .}$ | Display Anderson Theater problem. At Anderson theater if you <br> buy some tickets you get some free popcorn. Find the ratio <br> of popcorn to movie tickets. Display answer choice. | Selects ratio 8:3 |

## BUILDING ESSENTIAL UNDERSTANDING: Given a picture of an item with a constant ratio, select a picture representing the same ratio (Skip this section for students who understand these relationships and can identify these concepts)

INTRODUCE PROBLEM: Some stores offer frequent buyer cards.
MODEL THE PROCESS: Display "Chelsea" frequent buyer card. This says if you buy 6 coffees you get one free. The ratio is $6: 1$. I want to see if I can find another frequent buyer card with a similar deal. Display "Ralph's" and "Movie" cards. Here are two frequent buyer cards. Point to Ralph's. I will count to see if this card has the same ratio. Count the roller rink pictures with me. One. Two. Three. There are three roller rinks. Point to "free" And 1 free. If I skate at the roller rink 3 times I get 1 free visit. The ratio is 3:1. This is NOT the same. Point to the "movie" card. Let's see if this card has the same ratio. Count the movie pictures with me. One. Two. Three. Four. Five. Six. There are six movies. Point to "free" And 1 free. If I buy six movies. I get 1 free movie. The ratio is 6:1. This IS the same.

STUDENT PRACTICE: Now it's your turn to practice finding the same ratio. Use CONSTANT TIME DELAY script as needed to help students identify correct answer.
${ }^{* *}$ Note: Students may need repeated practice to learn this skill. Be sure to vary the order and placement of answers. Also, create your own frequent buyer cards using note cards, mini stickers and/or stamps to provide additional practice.
CHECK AND SCORE

| Step | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| 1. | Display "Rachael" card. This says if you buy 4 manicures you get one <br> free. The ratio is 4:1. Display video game and donut cards. Point to the <br> card that has the same ratio. 4:1. | Points to/Indicates "donut" card. |
| $\mathbf{2 .}$ | Display "Julie" card. This says if you buy 2 books you get one free. The <br> ratio is 2:1. Display "backpack" and "Nadia" cards. Point to the card that <br> has the same ratio. 2:1. | Points to/indicates "backpack" card. |

This may be a good stopping point. Have the student have a class room student election (they can vote on class president for a day or student of the day). Students can fill out ballets, turn them in, count out the results, and mark them on a table and create a bar graph. There is a generalization worksheet with this level. You can use this for additional guided practice or to send home as homework.

| INDEPENDENT <br> PRACTICE: <br> Ratio and <br> Proportion <br> Skills Test Teacher Says/Does  <br>  Give each student the Ratio and Proportion Skills Test: <br> Concepts and Symbols. Read directions for each problem <br> and have student select response. Record whether <br> response is correct or incorrect. Only provide praise for completing assessment (if student <br> needs encouragement). Do not provide specific praise for <br> correct answers while student is testing. <br>  NOW <br> Stop the lesson here and repeat tomorrow if student is not <br> yet getting at least 8 independent correct responses. Score <br> responses 1-14 on the Ratio and Proportion Progress <br> Monitoring Sheet if you did not do so while teaching. NEXT <br> Remember the goal is for students to be able to solve multi- <br> step proportional relationship problems, move into the second <br> half of the lesson to hit the target CCC for this grade level. You <br> can skip this Conceptual Foundation section to move on. |
| :--- | :--- | :--- |

## $6^{\text {th }}$ BUILD A GRADE ALIGNED COMPONENT: Write a ratio that matches a pictured ratio relationship $7^{\text {th }} \& 8^{\text {th }}$ SYMBOL USE: Fluency counting and review using standard notation for writing ratios

INTRODUCE THE ACTIVITY/PROBLEM: Remember we are learning about ratios and that many stores offer rewards or deals that use ratios. Now we will learn to write our own ratios. Display "Studious Schools Supplies" flyer. Here is a flyer for Studious Schools Supplies. This store is having a sale. If you buy some school items you get some free. Let's practice writing ratios for these deals.

- $\}$MODEL THE PROCESS: (Use the MODEL-LEAD-TEST script as a guide to teach this skill.) Point to the glue sticks and crayons section. Watch me as I write a ratio for this deal. To write a ratio you count the first set and write the number, write two dots, then count the second set and write the number. My turn to write the ratio. Point to crayons. l'll count these first. Point to each one as you count. One. Two. Three. Write three as you say, Now I write three. Write the colon as you say, Next I write two dots like this. Point to the crayon box. I count this second. Point to the crayon box as you count. One. Write one as you say, Now I write one. The ratio is three glue sticks to one box of crayons. I wrote 3 two dots then 1.

Give student a copy of the flyer. Point to the notebook and folders section. Write this ratio with me. Remember to write a ratio you count the first set and write the number, write two dots, then count the second set and write the number. Point to the folders. Count these with me first. Point to each one as you count making sure the student does the same. One. Two. Three. Four. Write four as you say, Write 4. Write the colon as you say, Write two dots like this. Point to the folders. I count these second. Point to each folder as you count. One. Two. Write two as you say, Write 2. The ratio is 4 binders to 2 folders. We wrote 4 two dots then 2.
$\leftrightarrow \xi$ STUDENT PRACTICE: Point to the remaining sections and say, Now it is your turn to write ratios for these deals. Use MODEL-LEAD-TEST script as needed to help students with each step (i.e., if student makes mistake model the solution, then lead them in finding it, then have them do it independently).
**Note: You may need to cut apart the student flyer if he/she has difficulty attending to the correct section. For students who are unable to count, have them use a number line or calculator (type one plus one, then push equals for each count after 2 and the calculator will "count" along for the student). If students are unable to write or type, have them indicate the numbers to write by using a speech output device or by eye gazing at the correct numbers and write for them.

CHECK AND SCORE

| STEP | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{3 .}$ | Point to the notebooks/highlighter section. Write the ratio for this <br> deal. | Student writes/indicates 5:5. |
| $\mathbf{4 .}$ | Point to the penci//eraser section. Write the ratio for this deal. | Student writes/indicates 7:3. |
| $\mathbf{5 .}$ | Point to the stapler/staples section. Write the ratio for this deal. | Student writes/indicates 1:2. |
| $\mathbf{6 .}$ | Point to the notebooks/calculator section. Write the ratio for this <br> deal. | Student writes/indicates 6:1. |

**Note: If students need further practice use notecards to create more "deals" or find advertisements in your local newspaper for students to use to practice writing ratios.


This may be a good stopping point. Have the student have a classroom student election (they can vote on class president for a day or student of the day). Students can fill out ballets, turn them in, count out the results, and mark them on a table and create a bar graph. They can calculate mean, median, mode, and range. There is a generalization worksheet with this level. You can use this for additional guided practice or to send home as homework.

| INDEPENDENT <br> PRACTICE: <br> Ratio and <br> Proportion <br> Skills Test | Teacher Says/Does |  |
| :--- | :--- | :--- |
|  | Give each student the Ratio and Proportion Skills Test: $6^{\text {rd }}$ <br> Grade Aligned Component. Read directions for each <br> problem and have student select response. Record <br> whether response is correct or incorrect. | Only provide praise for completing assessment (if student <br> needs encouragement). Do not provide specific praise for <br> correct answers while student is testing. |
|  | NOW <br> Stop the lesson here and repeat tomorrow if student is not <br> yet getting at least 3 independent correct responses. Score <br> responses 15-18 on the Ratio and Proportion Progress <br> Monitoring Sheet if you did not do so while teaching. | NEXT <br> Remember the goal is for students to be able to solve multi- <br> step proportional relationship problems as soon as possible, <br> move on to the next portion of the lesson to hit the target CCC <br> for the 7h grade level. |

## $7^{\text {th }}$ BUILD A GRADE ALIGNED COMPONENT: Given chart, calculate the proportional relationship between two items. $8^{\text {th }}$ SYMBOL USE: Review using calculator to divide to determine proportional relationship

INTRODUCE THE ACTIVITY/PROBLEM: Some ratios are the same all the time. Display faces chart. For example, all people have 2 eyes. Point to each person as you say, This person has two eyes. This person has two eyes and so does this person. Cover two of the faces. If l just have one person, the ratio is $2: 1$ Display $2: 1$ card. Uncover one face. If I have 2 faces the ratio is 4:2. Display 4:2. My ratio changed because I have more people. Point to each person's eyes as you say, But each person still has two eyes. If I have 3 people the ratio is 6:3. Display $6: 3$ card. My ratio changed because I have more people. Point to each person's eyes as you say, But each person still has 2 eyes. Gesture to each ratio. 6:3 , 4:2, and 2:1 are proportional. The proportional relationship is 2:1. Today we will find the proportional relationship for free offers at stores. Display "Hero Comics" chart. Hero Comics is having a sale. If you spend some money, you get some free comic books. Point across each corresponding row as you say, Socrates spent $\$ 24$ and received 2 free comic books. Ahmet spent $\$ 60$ and received 5 free comic books. And Pippa spent $\$ 36$ and received 3 free comic books. What is the proportional relationship between money spent and free comic books?

MODEL THE PROCESS: To find the proportional relationship, I want find out how much I need for just 1 item. The ratio is money spent to free comic books. I can find the proportional relationship by using the numbers from any row in my chart. I will use this row. Point to the Socrates row. Socrates spent $\$ 24$ and received 2 free comic books. Display proportional relationship graphic organizer (GO).


Point to blue square. First I write the bigger number here. The bigger number is $\$ \mathbf{2 4}$. Write 24 in the blue square. Point to green square on top row. Next I write the smaller number here. The smaller number is 2 . Write 2 in the green square on top row. Now I will divide both sides by the smaller number. I will write 2 in the green squares here. Point to green squares on second row and write 2 in each square. Display calculator. I will use my calculator to divide this side. Gesture to the left column. Push corresponding calculator buttons as you say, 24, first I push 2 then I push 4, divided by 2 equals 12 . Point the white square. I will write 12 here. Write 12. Point to the left side. This is 2 divided by 2. The rule is: when you divide a number by the same number the answer is always 1 . Point to the 1 on the GO. So 1 is already written here. The answer is 12 to 1 . If you spend $\$ 12$ at Hero comics you get 1 free comic.

STUDENT PRACTICE: Now it's your turn to find a proportional relationship. Display Super Subs Chart. Super Subs is having a promotion. If you buy some subs you get some free cookies. Point across each corresponding row as you say, The Jones family bought 3 sub sandwiches and got 6 free cookies. The Goldman family bought 4 sub sandwiches and got 8 free cookies. The Hirsh Family got 2 sub sandwiches and got 4 free cookies. What is the proportional relationship between sub sandwiches and free cookies? Display GO and calculator. Use these to help you find the answer. Use LEAST INTRUSIVE PROMPTS script as needed to help students with each step.
**Note:

- $\leqslant\}$ row with me, your turn to point to a row).
- When repeating this section, prompt students to find answer by using a different row each time.

CHECK AND SCORE
CHECK AND SCORE

| STEP | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{7 .}$ | Wait three seconds then say, Write the bigger number here. | Writes bigger number in blue square. |
| $\mathbf{8 .}$ | Wait three seconds then say, Write the smaller number here. | Writes smaller number in green squares. |


| 9. | Wait three seconds then say, Use your calculator to divide. | Uses calculator to divide bigger number by smaller number. |
| :---: | :--- | :--- |
| $\mathbf{1 0 .}$ | Wait three seconds then say, Write the answer on your <br> calculator here. | Writes solution in white box. |
| $\mathbf{1 1 .}$ | What is the proportional relationship? | Indicates answer. |

STUDENT PRACTICE: Your turn to do one more. Display Online Music Emporium Chart. Online Music Emporium is having a sale. If you buy some songs you get some free music videos. Point across each corresponding row as you say, Lukas bought 9 songs and received 3 free music videos. Jeremy bought 18 songs and received 6 free music videos. Matthew bought 12 songs and received 4 free music videos. What is the proportional relationship between songs purchased and free music videos? Display GO and calculator. Use these to help you find the answer. Use LEAST INTRUSIVE PROMPTS script as needed to help students with each step.
CHECK AND SCORE

| STEP | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{1 2 .}$ | Wait three seconds then say, Write the bigger number here. | Writes bigger number in blue square. |
| $\mathbf{1 3 .}$ | Wait three seconds then say, Write the smaller number here. | Writes smaller number in green squares. |
| $\mathbf{1 4 .}$ | Wait three seconds then say, Use your calculator to divide. | Uses calculator to divide bigger number by smaller number. |
| $\mathbf{1 5 .}$ | Wait three seconds then say, Write the answer on your <br> calculator here. | Writes solution in white box. |
| $\mathbf{1 6 .}$ | What is the proportional relationship? | Indicates answer. |

This may be a good stopping point. Have the student have a classroom student election (they can vote on class president for a day or student of the day). Students can fill out ballets, turn them in, count out the results, and mark them on a table and create a bar graph. They can calculate mean, median, mode, and range. There is a generalization worksheet with this level. You can use this for additional guided practice or to send home as homework.

| INDEPENDENT <br> PRACTICE: <br> Ratio and <br> Proportion <br> Skills Test | Teacher Says/Does |  |
| :--- | :--- | :--- |
|  | Give each student the Ratio and Proportion Skills Test: ${ }^{\text {Gh }}$ <br> Grade Aligned Component. Read directions for each <br> problem and have student select response. Record <br> whether response is correct or incorrect. | Only provide praise for completing assessment (if student <br> needs encouragement). Do not provide specific praise for <br> correct answers while student is testing. |
|  | NOW <br> Stop the lesson here and repeat tomorrow if student is not <br> yet getting at least 6 independent correct responses. Score <br> responses 19-28 on the Ratio and Proportion Progress <br> Monitoring Sheet if you did not do so while teaching. | NEXT <br> Remember the goal is for students to be able to solve multi- <br> step proportional relationship problems as soon as possible, <br> move on to the next portion of the lesson to hit the target CCC <br> for the 8 $8^{\text {th }}$ grade level. |

## $8^{\text {th }}$ BUILD A GRADE ALIGNED COMPONENT: Use graph to determine proportional relationship of $X$ and $Y$ value when $X$ equals 1 .

INTRODUCE ACTIVITY/PROBLEM: We have been learning about proportional relationships with store rewards using a chart and dividing. We can also identify a proportional relationship using a graph.

MODEL THE PROCESS: Display Acme Arcade graph. This is a graph for Acme Arcade. When you play games at the arcade you can get tickets. You can turn the tickets in and receive Better Bucks that can be used to buy prizes. This graph tells us how many tickets you need to get bucks. I want to know the proportional relationship of tickets to bucks. Watch as I find out how many tickets you need to get one buck. Point to the $x$-axis. This line tells how many Better bucks. Point to the $Y$-axis. This line tells me how many tickets. Put your finger on the origin $(0,0)$. Remember I want to know how many tickets you need to get one buck. First I put my finger here. Slide your finger along the $x$-axis to 1 . Then I slide it across the line until I get to the number one. Slide your finger vertically and stop at the blue line. Now I will slide my finger up until I get to the blue line. The blue line tells me about the tickets and bucks that I need. I stop on the blue line. My finger is above the one. Slide your finger to the left towards the $y$-axis and stop. Now I will slide my finger over to the ticket line and stop. My finger stopped at the number two. If I want one buck I need 2 tickets. The ratio is $1: 2.1$ buck to two tickets. Write the ratio on a dry erase board.

STUDENT PRACTICE: Now it's your turn to find a proportional relationship. Display the "Acme New Management Graph." Here is another chart. The Acme Arcade is under new management. They have changed the ratio of tickets to bucks. Find the proportional relationship. How many tickets to get 1 buck? Use LEAST INTRUSIVE PROMPTS script as needed to help students with each step.
${ }^{* *}$ Note: If a student is unable physically trace the graph, you could do it for them and ask them to tell you to stop at the correct places on the chart.

[^1]| CHECK AND SCORE |
| :--- |
| Step Teacher Says/Does Student Response <br> $\mathbf{1 7 .}$ Wait three seconds then say, Put your finger on the 1 on the <br> Better Bucks line. Puts finger on 1 on x-axis. <br> $\mathbf{1 8 .}$ Wait three seconds then say, Move your finger up and stop at <br> the blue line. Slides finger up until reaches blue line. <br> $\mathbf{1 9 .}$ Wait three seconds then say, Slide your finger over to the ticket <br> line. Slides finger over to $y$-axis. <br> $\mathbf{2 0 .}$ Wait three seconds then say, What number did you stop at? Indicates 3. <br> $\mathbf{2 1 .}$ What is the ratio of bucks to tickets? Indicates 3:1 |

## $8^{\text {th }}$ BUILD A GRADE ALIGNED COMPONENT: Solves two-step ratio problem using manipulatives.

INTRODUCE ACTIVITY/PROBLEM: We learned about the ratio of bucks to tickets at the Acme Arcade. The reason we want tickets is so that we can buy prizes. Display the Acme prize chart. I want to know how many tickets I will need to buy a prize.

MODEL THE PROCESS: Today I want to buy the bouncy ball**. Point to the key below the chart. Remember, the ratio is $\mathbf{2}$ tickets to $\mathbf{1}$ better buck. Point to the bouncy ball row on the chart. It says here that I need 5 better bucks to buy the bouncy ball. How many tickets to I need to buy the bouncy ball? Display better buck graphic organizer (see example below) and stop sign. I can use this organizer to help me. Each row has one buck on it and all the rows are numbered. Gesture to the chart. The chart says I need 5 bucks to buy the bouncy ball. Point to the $5^{\text {th }}$ buck on the GO. This is buck number five. I only need 5 bucks sol will put my stop sign below to remind me to stop at the $5^{\text {th }}$ better buck. Display the tickets (more than you will need) and point the key on the chart. Remember the ratio is $\mathbf{2}$ tickets to $\mathbf{1}$ better buck. Place the tickets as you say, For each buck on my chart I will put two tickets like this. One, two. One, two. Continue counting to 2 and placing the tickets on the chart until you get to the $5^{\text {th }}$ buck. Now I have 5 bucks and 2 tickets for each buck. Display all together graphic organizer. I need to know how many tickets I have all together. Slide the tickets onto the top of the mat as you say. I will slide all these tickets onto my all together mat like this. Slide each ticket across the line as you count. No I can count my tickets. One. Two. . . ten. There are ten tickets in all. I need ten tickets to buy the bouncy ball.


O
STUDENT PRACTICE: Now it's your turn. Display the Acme New Management Chart. The Acme Arcade is under new management. Here is the chart telling the prizes you can buy and how much they cost. Point to the key chain. How many tickets do you need to buy the key chain? Point to the key. Remember, the ratio is 3 tickets to 1 better buck. Provide the student with tickets, the buck GO, stop sign, and the all together mat. Use these to find out how many tickets you will need to buy the keychain. Use LEAST INTRUSIVE PROMPTS script as needed to help students with each step.
**Note: Change the prize you buy and they buy each day to ensure students are not memorizing the answers. IF students are not able to count with one to one correspondence, provide them with a number chart or number line. They can place each ticket on a number and point to the correct number to identify the answer.
CHECK AND SCORE

| Step | Teacher Says/Does | Student Response |
| :---: | :--- | :--- |
| $\mathbf{2 2 .}$ | Wait three seconds then point to the four beside the keychain. It <br> says you need 4 bucks, put the stop sign under the fourth buck <br> on your chart. | Puts stop sign below $4^{\text {th }}$ buck on GO. |
| $\mathbf{2 3 .}$ | Wait three seconds then say, Put three tickets beside each buck. <br> Stop when you get to the fourth buck. | Puts three tickets beside each buck. |
| $\mathbf{2 4 .}$ | Wait three seconds then say, Slide the tickets onto the all <br> together mat. | Slides tickets onto all together mat. |
| $\mathbf{2 5 .}$ | Wait three seconds then say, Count the tickets | Counts out the tickets. |
| $\mathbf{2 6 .}$ | How many tickets do you need to buy the keychain? | Answers 12. |

This is the end of the lesson. Have the student have a classroom student election (they can vote on class president for a day or student of the day). Students can fill out ballets, turn them in, count out the results, and mark them on a table and create a bar graph. They can each campaign in the days ahead and track how many hours they spent campaigning. They can analyze the relationship between hours spent campaigning and votes received. There is a generalization worksheet with this level. You can use this for additional guided practice or to send home as homework.

| INDEPENDENT <br> PRACTICE: Ratio <br> and Proportion <br> Skills Test | Teacher Says/Does |  |
| :--- | :--- | :--- |
|  | Give student the Ratio and Proportion Skills Test: $8^{\text {th }}$ grade <br> aligned component. <br> Read directions for each problem and have student <br> select response. Record whether response is correct or <br> incorrect. | Only provide praise for completing assessment (if student <br> needs encouragement). Do not provide specific praise for <br> correct answers while student is testing. |

## Troubleshooting and Data-Based Decision Making for Ratio and Proportion Skills Test:

If student is unable to complete any items on the Ratio and Proportion test independently and correctly, go back and teach one problem step-bystep.

MASSI CULMINATING ACTIVITY: Go to the local mall, shopping center, or online, if you live in a remote area, and have students visit stores to ask about rewards programs and or frequent buyers cards. When you return to school have them develop ratios representing the store rewards. Have them calculate the rewards in dollars as wells as items (e.g., 4 t -shirts: 1 free t:shirt or $\$ 100$ spent: $\$ 25$ free).

## BUILD TOWARDS FULL GRADE LEVEL COMPETENCE

Here are ideas to build competence towards the full grade level competence using this same activity. See the unit plan and talk with the general education teacher for more ideas.

| Component | Activity | What Student Does | Generalization/Fluency |
| :--- | :--- | :--- | :--- |
| Compare two ratios to determine <br> whether they are proportional | Provide students with frequent <br> buyer cards, some that are <br> proportional, some that are not | Determine the constant of <br> proportionality (unit rate) of each <br> ratio and compare to the other <br> ratios to see if they are the same <br> (thus proportional) | Repeated practice with comparing <br> ratios to determine proportionality |

## Worksheet 1: Essential Understandings



## Worksheet 2: $6^{\text {th }}$ Grade Aligned Component



## Worksheet 3: $7^{\text {th }}$ Grade Aligned Component



## Worksheet 4: $8^{\text {th }}$ Grade Aligned Component

| What is the proportional relationship shown on this graph? Circle the ratio. | What is the proportional relationship shown on this graph? Circle the ratio. |
| :---: | :---: |
| 2 tickets $=1$ better buck 3 better bucks = beach ball How many tickets to buy a beach ball? <br> Draw or use manipulatives to solve. Circle your answer. <br> 6 tickets <br> 5 tickets <br> 12 tickets | 4 tickets $=1$ better buck <br> 5 better bucks = remote controlled car <br> How many tickets to buy a remote controlled car? <br> Draw or use manipulatives to solve. Circle your answer. <br> 9 tickets <br> 20 tickets <br> 10 tickets |

## Materials:



| 4:2 <br> 4 movie tickets to 2 popcorns |  |
| :---: | :---: |
| 5:5 <br> 5 movie tickets to 5 popcorns | 4:1 <br> 4 movie tickets to 1 popcorn |



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Studious School Supplies: Back to School Sale

|  |  |
| :---: | :---: |
|  | Get <br> Free |
|  |  |

## Hero Comics

Money Spent Free Comic Books

| Socrates | $\$ 24$ | 2 |
| ---: | ---: | :--- |
| Ahmet | $\$ 60$ | 5 |
| Pippa | $\$ 36$ | 3 |

What is the proportional relationship between money spent and free comic books?

## Super Subs!

|  | Sub Sandwiches | Free Cookie |
| ---: | :---: | :---: |
| Jones Family | 3 | 6 |
| Goldman Family | 4 | 8 |
| Hirsh Family | 2 | 4 |

What is the proportional relationship between sub sandwiches and free cookies?

|  |  |  |
| :---: | :---: | :---: |
|  | Songs Purchased | Free Music Video Download |
| Lukas | 9 | 3 |
| Jeremy | 18 | 6 |
| Matthew | 12 | 4 |

What is the proportional relationship between songs purchased and free music video downloads?


## Proportional Relationship Graphic Organizer






| Acme Arcade |  |
| :---: | :---: |
| Better Bucks | Prize |
| 5 | Bouncy ball |
| 10 | Stuffed animal |
| 15 | Acme T-shirt |

2 tickets = 1 Better buck How many tickets to buy a prize?


| $1{\underset{1}{1} \text { ONEDOLLAR }}_{1}^{\$}{ }^{1}$ |
| :---: |
| $2 \operatorname{cic}_{1}^{1} \underset{\text { ONE DOLLAR }}{\$ 1}$ |
| $3{ }_{1}^{1}$ M ONEDOLLAR ${ }^{1} 1$ |
| 4 <br>  |
| $5 \underset{1}{1} \text { a ONEDOLLAR }{ }^{1}$ |
|  |
| 7 $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ |
| $8{\underset{1}{1} \underset{\text { ONEDOLLAR }}{1} 1}_{\$}^{\text {ONE }}$ |
| $9 \cos _{1}^{1} \underset{\text { ONEDOLLAR }}{\$ 1}$ |
| $10^{1} \text { y } \underset{1}{\text { ONEDOLLAR }} \$ 1$ |
|  |
| 12 H. ONE $\square$ |
| $13^{1} \text { H } \${ }_{\text {ONE }}{ }^{1}$ |
|  |
|  |




[^0]:    ${ }^{1}$ The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

[^1]:    MASSI: Middle School Ratio and Proportion, July 2013

