Mathematics:

Measurement

Middle School

NCSC Sample Instructional Unit

Universal Design for Learning (UDL) Instructional Unit – Surface Area and Volume

National Center and State Collaborative

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

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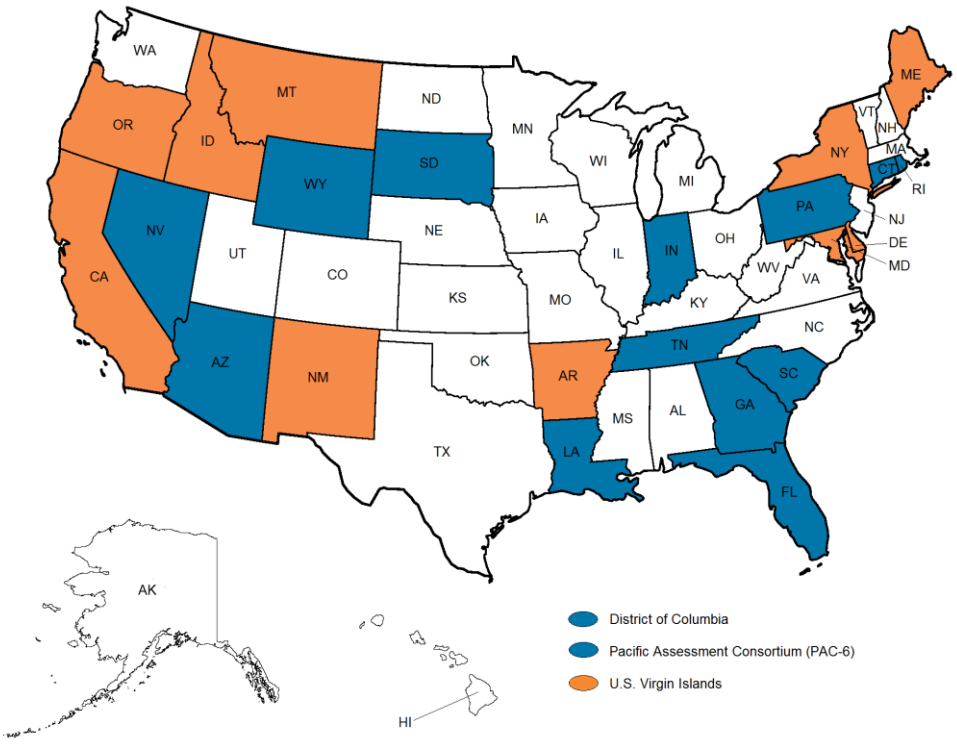
This document is available in alternative formats upon request.

National Center and State Collaborative

NCSC is a collaborative of 15 states and five organizations.

The states include (shown in blue on map): Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, Montana, New Mexico, New York, Oregon, and U.S. Virgin Islands.



*Core partner states are blue in color and Tier II states are orange in color.

[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

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Unit Key Vocabulary

Acute triangle – a triangle where all three angles are acute (less than 90°)

Area – the measure of the size of the surface of a two-dimensional figure; measured in square units

Base – side of a plane figure or face of a solid, particularly one perpendicular to the direction height is measured

Complex shape – a figure that includes the elements of two or more simple shapes (triangles, rectangles, squares, circles)

Cubic centimeter – the volume equal to a cube one centimeter on each side

Cubic foot – the volume equal to a cube one foot on each side

Cubic inch – the volume equal to a cube one inch on each side

Cubic unit – a unit of measurement of volume or capacity

Edge – a line segment joining two adjacent vertices in a two-dimensional figure or a shared boundary between two faces of a three-dimensional figure

Equilateral triangle – triangle in which all three sides are equal in length

Face – a flat surface of a three dimensional figure

Height – the perpendicular distance *of a two- or three- dimensional shape* from the base to the opposite vertex

Length – a measurement of the distance from one point to another

Net – a flattened three-dimensional figure that can be turned into a solid by folding it

Obtuse triangle – a triangle that has one internal angle greater than 90°

Quadrilateral – a polygon that has four sides

Parallel lines – two lines that lie in the same plane and maintain the same distance apart over their entire length, and never intersect

Rectangle – a quadrilateral (four-sided figure) with four right angles

Rectangular prism – a three-dimensional figure of which all faces are rectangles

Right angle – an angle of exactly 90°

Right triangle – a triangle containing an angle of 90°

Square – a rectangle with four congruent sides

Square foot – the area equal to a square one foot on each side

Square inch – the area equal to a square one inch on each side

Square meter – the area equal to a square one meter on each side

Square unit – a unit of measurement of area

Surface area – the total area of all the faces or surfaces of a three-dimensional figure

Trapezoid – a quadrilateral that has exactly one pair of parallel sides

Vertices – plural for vertex, the common point where two sides or edges meet

Volume – a measure of the number of cubic units needed to fill the space inside an object

Width – the minimum distance between parallel lines of a figure

Unit Standards Overview

Common Core State Standard:
7.G Geometry **7.G.B** Solve real-life and mathematical problems involving angle measure, area, surface, area, and volume. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms
6.G Geometry **6.G.A** Solve real-world and mathematical problems involving area, surface, area, and volume. **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems
5.MD Measurement & Data **5.MD.C** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.C.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **5.MD.C.3a** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **5.MD.C.3b** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.C.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.C.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. **5.MD.C.5a** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. **5.MD.C.5b** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

5.MD.C.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems

5.G Geometry

5.G.B Classify two-dimensional figures into categories based on their properties.

5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Common Core State Standard: Standards for Mathematical Practice

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.
CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
CCSS.Math.Practice.MP4 Model with mathematics.
CCSS.Math.Practice.MP5 Use appropriate tools strategically.
CCSS.Math.Practice.MP6 Attend to precision.
CCSS.Math.Practice.MP7 Look for and make use of structure.
CCSS.Math.Practice.MP8 Look for and express regularity in repeated reasoning.

Learning Progressions Frameworks Progress Indicator

M.ME.1a identifying and describing measurable attributes (including area, surface area, volume, fractional units, absolute value with temperature), and selecting appropriate customary or metric units of measure when solving problems
M.ME.1c recognizing how the formulas for area and volume for a variety of shapes and solids are related
M.ME.2a selecting and applying appropriate standard units, tools, and level of precision in real-world measurement problems (e.g., area, surface area, volume, rate)
M.ME.2b using a variety of strategies (decomposing complex shapes, using formulas and models) to measure area (triangles, quadrilaterals, polygons) and volume (rectangular prisms)
M.ME.2c selecting and applying appropriate standard units and tools to measure to an appropriate level of precision
M.GM.1a describing and classifying plane figures based on their properties
M.GM.1d solving area, surface area, and volume problems by composing and decomposing figures
M.GM.1h solving real-world area, surface area, and volume problems using different strategies (formulas and decomposing figures)

Instructional Family: Geometric Problems

CCCs addressed:

- **7.GM.1h1** Add the area of each face of a prism to find surface area of three dimensional objects
- **7.GM.1h2** Find the surface area of three-dimensional figures using nets of rectangles or triangles
- **7.GM.1h3** Find area of plane figures and surface area of solid figures (quadrilaterals)
- **7.GM.1h4** Find area of an equilateral, isosceles, and scalene triangle
- **6.GM.1d1** Find area of quadrilaterals
- **6.GM.1d2** Find area of triangles

Instructional Family: Recognizing, Describing, Naming and Classifying

CCCs addressed:

- **5.GM.1a1** Recognize properties of simple plane figures
- 4.GM.1h2 Classify two--dimensional shapes based on attributes (# of angles)
- 2.GM.1b3 Distinguish two- or three- dimensional shapes based upon their attributes (i.e., #of sides, equal or different lengths of sides, # of faces, # of corners)

Instructional Family: Perimeter, Area, Volume

CCCs addressed:

- **7.ME.2c1** Solve one step real world measurement problems involving area, volume, or surface area of two- and three-dimensional objects
- **6.ME.2a3** Apply the formula to find the area of triangles
- **6.ME.2b2** Decompose complex shapes (polygon, trapezoid, pentagon) into simple shapes (rectangles, squares, triangles) to measure area
- **6.ME.1c1** Find the area of a 2-dimensional figure and the volume of a 3-dimensional figure
  6.ME.1a2 Identify the appropriate formula (i.e., perimeter, area, volume) to use when measuring for different purposes in a real life context
- **5.ME.2b1** Use filling and multiplication to determine volume
- **5.ME.2b2** Apply formula to solve one step problems involving volume

Mathematics: Geometry Unit

Lesson #1

Objective

- Students will solve real-life and mathematical problems involving area, surface area, and volume.

Essential Questions

- How do we find the area of a square, rectangle, right triangle, and triangle?

Vocabulary

Acute triangle	Rectangle
Area	Right angle
Base	Right triangle
Edge	Square
Equilateral triangle	Square foot
Face	Square inch
Height	Square meter
Length	Square unit
Obtuse triangle	Trapezoid
Quadrilateral	Vertices
Parallel lines	Width

Materials

Blank grid paper so students can draw:

- A rectangle with the dimensions of 5 x 3
- A right triangle with sides measuring 3 x 5 x 6
- A square measuring 5 x 5

Calculator

Chart to be used for practice to reinforce skills.

Example of square inch

Example of square foot

Example of square meter

Formula sheet for the area of rectangles, squares, and triangles:

- Area of rectangles/squares A = l x w
- Area of triangles A = ½ b x h

Grid paper or a geoboard to provide students with the following drawn shapes:

- Square (2 x 2)
- Rectangle (3 x 7)
- Right triangle
- Acute triangle
- Obtuse triangle

Paper and pencils

Shape attribute blocks

Unit tiles

Worksheets

- Lesson 1 Body
- Lesson 1 Practice

Introduction

a. Activate Previous Knowledge

1) Lead a short discussion about the attributes of common shapes. During the discussion, have the students locate these shapes in their environment.
   a) Review the attributes of common shapes:
      - A square has 4 equal sides with opposite sides parallel and 4 right angles.
      - A rectangle has 4 sides with opposite sides equal and parallel and 4 right angles.
      - A triangle has 3 sides.
      - A right triangle has 3 sides and one right angle.
2) Break the class into small groups to answer exercises:
   a) Using figures (rectangle, square, triangle and right triangle) drawn on grid paper or formed on geoboards, have the students identify the shapes.
   b) Using given dimensions for lengths of sides and the angles of particular shapes, have students draw the shapes on grid paper or form on geoboards.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc., to complete the exercises. Present information within the context of student interests (pets, gardening, new bedroom floor plan, etc.)

Additional Considerations for Emerging Readers and Emerging Communicators

a. Activate Previous Knowledge

1) Lead a short discussion about the attributes of common shapes. During the discussion, have the students locate these shapes in their environment.
   a) Provide graphic and/or digital representations of the four common shapes.
      - Digital representations may include animations with sound to reinforce critical attributes.

- Graphic representations may include tactile and additional visual qualities, such as a purple right triangle with visually or tactilely represented numerals (1-3) marked on each side and the right angle indicated with an arrow labeled "90°".
- Graphic organizers may show shapes and critical attributes/details.
- Graphic representations may include tactile (outlines in glue, yarn, etc.) and additional visual qualities (color, pattern, etc.).
- Pre-teach concepts that the student may not have fully acquired. Use systematic instruction techniques as appropriate.

b) Provide photos of environmental shapes with or without visual or tactile outlines.

- Provide photos on digital display with or without speech generation.
- Preplan a shape for the student to contribute to the discussion. Allow the student to use his/her preferred mode of communication (verbal/vocal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, tactile symbols, AAC, etc.). Preplanning could include:
  - raised line grid paper;
  - enlarged cell/scale grid paper;
  - outlined shapes with glue, yarn, etc.;
  - student using representations, organizers, etc. from 1.a. to identify shapes;
  - match shapes; and/or
  - cut out grid paper figures and allowing the student to sort them.

2) Break the class into small groups to answer exercises:

a) Allow student to use representations, organizers, etc. from 1.a. to identify shapes.

b) Have the student measure shapes drawn on paper or cut from foam-board, and identify which one has the correct dimensions.

- Use adaptive measuring tools (such as a ruler with tactile qualities or a piece of paper laminated on card stock substituting for the paper shape if the student needs those; other rulers that accommodate the student more effectively may be used, such as:
  - digital rulers, bendable/foldable rulers,
  - tactile rulers,
  - transparent/translucent rulers,

  - simplified rulers with only inches or centimeters marked,
  - rulers with hook-and-loop tape or other "handles") as necessary.
- After the student identifies a shape, have him/her trace the shape onto grid paper.
- Select digital shape from several options and drag to digital grid or geoboard. Add sound to digital activity.
- Have the student feel shapes created by partners on geoboard.
- Provide multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.
- Provide multiple means of expression**:** Allow students to use paper and pencil, models, manipulatives, computers, etc., to complete exercises.
- Provide multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc.

b. Establish Goals/Objectives for the Lesson

Inform students that in this lesson they will make decisions as to the appropriate measurements and formulas to use in solving real world and mathematical problems involving the area of common shapes.

Explain that they will:

1. Describe and understand what area measures and the correct units for area measures.
2. Determine the area of rectangles and squares using graph paper/manipulatives and formulas.
3. Determine the area of right triangles using graph paper/manipulatives and formulas.
4. Determine the area of other triangles using graph paper/manipulatives and formulas.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journal, computer, graphic organizers (premade or original), etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me".

Additional Considerations for Emerging Readers and Emerging Communicators

A. Provide the definition and unit measures for area with or without symbol-based text, with textures, other tactile qualities, colors, etc. (resources such as Standard Tactile Symbol List available from the Texas School for the Blind www.tsbvi.edu/tactile-symbols may be useful in determining representations; some representations may need to be pre-taught (Rowland, 2012), but use of the same representations/symbols [or the system] during this and other lessons will reduce the need for pre-teaching).

B. Provide definitions and images of squares and rectangles with or without symbol-based text, with textures, other tactile qualities, colors, etc. Provide:

- formulas and examples of calculating area of squares and rectangles with or without symbol-based text with textures, other tactile qualities, colors, etc.
- calculator (large button, handheld or computer-based).

C. Provide the definition and images of right triangles with or without symbol-based text, with textures, other tactile qualities, colors, etc. Provide:

- formula and examples of calculating area of right triangles with or without symbol-based text, with textures, other tactile qualities, colors, etc.
- calculator (large button, handheld or computer-based).

D. Provide the definition and images of other triangles with or without symbol-based text, with textures, other tactile qualities, colors, etc. Provide:

- formula and examples of calculating area of other triangles with or without symbol-based text, with textures, other tactile qualities, colors, etc.
- a calculator (large button, handheld or computer-based).

Body

1. Lead a discussion about what area is (the size of a surface; the amount of space inside the boundary of a flat, 2-dimensional shape such as a rectangle; the amount of carpet needed to cover the floor).
    - Ask the students:
        - What units (square units, square inches, square feet, square meters) are used to measure area? Use Lesson 1 Body worksheet, page 1.

2. Find the area of a rectangle and a square using grid paper/manipulatives and formulas. Use Lesson 1 Body worksheet, page 2.
    - Working with rectangles, ask the students:
        - What is the area of the rectangle by counting units?
        - What is the length of the rectangle?
        - What is the width of the rectangle?
        - Show the students that they can find the area of the rectangle by multiplying length times width, e.g., 4 x 5 = 20 units^2.
    - Working with squares, ask the students:
        - What is the area of the square by counting units?
        - What is the length of the square?
        - What is the width of the square?
        - How do the length and width compare in a square?
        - Show the students that they can find the area of the square by multiplying length times width, e.g., 5 x 5 = 25 units^2.
        - Ask the students: What would be the area of Sarah's rectangular garden if it has a length of 12 feet and a width of 3 feet?
3. Find the area of a right triangle using grid paper/manipulatives and formulas. Use Lesson 1 Body worksheet, page 3.
    - Ask the students: "What is the area of the right triangle by counting units?"
    - Demonstrate to the students how to use the formula for the area of a right triangle: (Area = ½ base times height). (Draw a square and find its area. Draw a diagonal through the square to form 2 right triangles. Each triangle is one-half the area of the square.)
    - Ask the students:
        - What is the height of the right triangle?
        - What is the base of the right triangle?

4. Find the area of a triangle (not a right triangle) using grid paper/manipulatives and formulas. Use Lesson 1 Body worksheet, page 4.
    - Show students the base and height of a triangle by using a right triangle and triangles that are not right triangles.
    - Ask the students:
        - Can any side of a triangle be the height and base?
        - What is common about the height and base in the two triangles?
        - What is the measure of the base of the triangle?
        - What is the measure of the height of the triangle?
        - What is the area of the triangle?

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc., to complete the exercises. Present information within the context of student interests (pets, gardening, new bedroom floor plan, etc.)

Additional Considerations for Emerging Readers and Emerging Communicators

1) Demonstrate the definition and unit measures for area with or without symbol-based text, with textures, other tactile qualities, colors, etc. (resources such as Standard Tactile- Symbol List available from the Texas School for the Blind www.tsbvi.edu/). Tactile-symbols may be useful in determining representations; some representations may need to be pre-taught (Rowland, 2012), but use of the same representations/symbols [or the system] during this and other lessons will reduce the need for pre-teaching. Provide:
    - choices from which the student can select. Depending upon the student's current understanding, distractors (incorrect answers) may be included. However, since this is instructional, the errorless learning technique in which all the choices would be correct is also appropriate;
    - formulas and examples of calculating area of squares and rectangles with or without symbol-based text with textures, other tactile qualities, colors, etc.; and/or

- rectangle and square shapes on grid paper, geoboard, or digitally (use virtual manipulatives such as those from National Library of Virtual Manipulatives) -
    - digital representations may include animations with sound to reinforce critical attributes. Graphic representations may include tactile and additional visual qualities.
    - manipulatives can be used to mark each unit in the shapes.

2) Working with rectangles and/or squares, have the student either count each unit on the representation or place a manipulative on each unit and then count the manipulatives for the first three questions.
    - Have the student count by using the "+1" function on the calculator (big button, hand-held or computer based) as he/she moves each manipulative.
    - Place manipulatives on a number line to count.
    - Provide a manipulative (string, pipe cleaner, strip of card stock paper) that is the length of the square and one that is the width of the square. Have the student compare the two manipulatives for the fourth question.
    - Allow the student to use his/her preferred mode of communication (verbal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, etc.).
    - Allow the student to use a calculator for the last question.
3) Provide the definition and unit measures for area with or without symbol-based text, with textures, other tactile qualities, colors, etc. (resources such as Standard Tactile Symbol List available from the Texas School for the Blind www.tsbvi.edu/tactile-symbols may be useful in determining representations; some representations may need to be pre-taught (Rowland, 2012), but use of the same representations/symbols [or the system] during this and other lessons will reduce the need for pre-teaching). Provide:
    - choices from which the student can select. Depending upon the student's current understanding, distractors (incorrect answers) may be included. However, since this is instructional, the errorless learning technique in which all the choices would be correct is also appropriate;
    - formulas and examples of calculating area of triangles with or without symbol-based text with textures, other tactile qualities, colors, etc. ;
    - rectangles made from two triangles (which can be separated and put back together) so the student can better understand the concept of the area formula;

- the triangle shapes on grid paper, geoboard, or digitally (use virtual manipulatives such as those from National Library of Virtual Manipulatives). Digital representations may include animations with sound to reinforce critical attributes. Graphic representations may include tactile and additional visual qualities. Provide manipulatives that can be used to mark each unit in the shapes.

4) Have the student either count each unit on the representation or place a manipulative on each unit and then count the manipulatives for the first three questions (area height, base). For the last set of questions:

- Provide the same triangle representations with which the student has been working.
- Provide manipulatives corresponding to the height and base of both triangles.
- Use adaptive measuring tools (such as a ruler with tactile qualities or a piece of paper laminated on cardstock substituting for the paper shape if the student needs those; other rulers that accommodate the student more effectively can be used such as digital rulers, bendable/foldable rulers, tactile rulers, transparent/translucent rulers, simplified rulers with only inches or centimeters marked, rulers with hook-and-loop tape or other "handles") as necessary.
- Allow the student to use his/her preferred mode of communication (verbal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, etc.).
- Use calculator.

Practice

1) In small groups, have students work on a variety of problems involving area, some using different shapes (rectangles, squares, right triangles, and triangles) and some using dimensions only to determine area. Use Lesson 1 Practice worksheet.

2) Give each student the chart below (page 1 of worksheet). Students may use models to explore the various possibilities and complete the chart.

Problem	Figure	Formula	Formula with values	Area (unit^2)
1	Rectangle	A = l x w	A = 2 x 7	14 units^2
2				
3				
4				
5				
6				
7				
8				
9				

3) Bring the whole group back together. Ask one student from each group to discuss the results of a specific problem. Complete the chart and discuss any problems they may have had.

Multiple means of representation: Provide students with a copy of the problems, formulas, and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow the students to solve the problem using formulas and/or models.

Multiple means of engagement: Ensure students are actively involved in their small groups. Present different problems related to student interests. As you observe group work, use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Ensure that the student participates in all activities in the group – selecting problems, solving problems, contributing to discussions, etc.
    - Allow the student to use his/her preferred mode of communication (verbal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, etc.). Other students will need to understand the student's communication mode and should be encouraged to use it themselves, serving as communication models/partners.
2) Have the student solve the problems using any accommodations that have been successful or shown promise in previous activities (models, definitions, formulas, steps, tactile/visual cues, graphics (2- or 3- dimensional, digital, manipulatives, calculators, adapted measuring tools, etc.). It is important to maintain consistency in accommodations and supports, providing and using the ones with which the student has the most success.
    - Since this practice is instructional, guidance in completing the chart is acceptable and expected. Errorless learning techniques and systematic instructional techniques may be helpful to use.
    - Reduce the number of problems.
    - Reduce the difficulty of some problems (e.g., focus on those with single digits only).
3) Preplan a problem for the student to present.
    - Allow the student to use his/her preferred mode of communication (verbal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, etc.).

Closure

d. Revisit/Review Lesson and Objectives

Remind students of the lesson's objectives and expected student outcomes. Have students discuss which of the lesson's problems addressed the objectives below and what they learned from their experiences.

Students will solve real life and mathematical problems involving area, surface area, and volume.

1) Describe and understand what area represents and the correct units of measure for area.
2) Determine the area of rectangles and squares using graph paper/manipulatives and formulas.
3) Determine the area of right triangles using graph paper/manipulatives and formulas.
4) Determine the area of other triangles using graph paper/manipulatives and formulas.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats: writing, drawing, creative expression, discussion, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

Provide all materials and accommodations used throughout the lesson and use them to remind the student as the teacher reviews.

Provide a graphic organizer (enhanced with additional tactile and visual qualities as appropriate) with categories of rectangles, squares, right triangles, and other triangles. Have the student sort (with guidance as necessary) characteristics, formulas, examples, problems, etc. into the categories (these may be symbolic, concrete, or a combination of both). Allow the student to use this to contribute to the discussion.

Provide several choices the student can select from to answer the question(s).

Preplan a contribution.

Allow the student to use his/her preferred mode of communication (verbal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, etc.)

Exit Assessment

Give students a new area problem:

Mr. Hanks is choosing between laminate and tile for his floor and wants to use the material that has the largest area per piece so he can get the job done faster with fewer materials. The laminate comes in rectangular strips that are 36 inches by 8 inches. The tiles are 18 inches by 18 inches. Find the area of each piece to decide which one Mr. Hanks should buy.

Have students work independently to find the area. Students should use a similar table as was used during Practice.

Multiple means of representation: Provide students with a copy of the work problem and the table. Have drawings and manipulatives available for the students to use.

Multiple means of expression: Allow the students to solve the problem using formulas and/or models.

Multiple means of engagement: Present different problems related to student interests. As you observe students working, use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide the problem with symbol-based text or text enhanced with tactile and visual qualities as necessary.
    - Provide a photo related to the text.
    - Have student highlight (color, underlined in glue, etc.) important parts of the text.
2) Provide graphic representations of materials (scaled, actual size, or actual samples of flooring materials).
    - Provide any accommodations or supports used successfully in other activities in the lesson (formulas, calculator, manipulatives, definitions, graphics, etc.).
    - Provide two graphic or digital floor plans divided into grids, one representing the laminate pieces and the other representing the tile pieces. Provide manipulatives for the student to place and count.
    - Provide answer choices for the student to choose from.
    - Simplify the language. For example, "…*he can get the job done faster with fewer materials.*" could be changed to "*…he can use a smaller number of pieces*."

As this is now assessment as opposed to all previous activities in this lesson, the student's first independent attempts solving the two problems and answering the question should be observed and recorded for accuracy.

Mathematics Geometry Unit

Lesson #2

Objective

- Students will solve real-life and mathematical problems involving area, surface area, and volume.

Essential Questions

- How can we find the area of quadrilaterals and complex shapes?

Vocabulary

Area	Quadrilateral
Base	Rectangle
Complex shape	Right triangle
Height	Trapezoid
Length	Width

Materials

Chart to be used for practice reinforcing skills.

Formula sheet for the area of rectangles, squares, and triangle:

- Area of rectangles A = l x w
- Area of triangles: A = ½ b x h

Geoboard

Grid paper

Square tiles

Worksheets

- Lesson 2 Intro 2
- Lesson 2 Intro 3
- Lesson 2 Body
- Lesson 2 Practice

Introduction

A. Activate Previous Knowledge

1. Remind students that they previously determined the areas of shapes such as rectangles, squares, and triangles.
    - Discuss with students that not all shapes or spaces are rectangles, squares, or triangles.
    - Ask the students: What are some shapes you see around you that are not rectangles, squares, or triangles?
    - What are some ways you might find the area of these quadrilaterals and complex shapes?
2. Lead a discussion about how to determine lengths from given dimensions and that lengths can be additive. Use Lesson 2 Intro 2 worksheet.
3. Provide practice worksheet on partitioning shapes and finding lengths. Use Lesson 2 Intro 3 worksheet.

Multiple means of representation: Provide drawings or models of partitioned shapes, shapes with some lengths and widths identified and lines with some measurements identified. Worksheets can be presented with fewer items per page.

- Worksheets may be printed on various colored sheets
- Worksheets may be presented on computer.

Multiple means of expression: Students may describe a situation where they have seen a shape partitioned or only knew some of the dimensions of a figure. Students may use drawings or models to complete worksheets. Students may complete work on a computer.

Multiple means of engagement: Provide examples of partitioned shapes and situations where they must determine measurements based on the dimensions they know based on interests of the students or real life situations in which they would be used.

Additional Considerations for Emerging Readers and Emerging Communicators

If the student has had little to no experience with the concept of area or the skill of calculating area, it might be helpful to provide instruction using the ***Lesson 2 Concept Reinforcement Activity (CRA)*** *prior to teaching the Introduction to Lesson 2.*

1. Provide the accommodations from Lesson 1 to remind students of shapes discussed and their details (length, width, base, height, angles, sides, area formulas, characteristics, etc.).

Provide representations (graphic or digital) of other shapes (trapezoids, quadrilaterals, complex shapes) found in the environment and examples of them (photos, icons, etc.).

- Enhance representations with tactile, visual, auditory qualities as necessary.

Preplan a shape of the student's choice to contribute to the discussion.

- Allow the student to use his/her preferred mode of communication (verbal, sign, writing, drawing, typing/keyboarding, speech generating device, symbols, etc.).

2. Provide the worksheet of a complex shape to each student and using the same shape/dimensions on the white board, smart board or computer projection, point out the different sides where the measurement is noted and sides without measurements.
    - Provide complex shapes composed of cut-outs of simple shapes.
        - Provide a model of complex shapes that the student can use a "puzzle" template to construct a complex shape from end to end to see how the side lengths are additive. Or use cut-out equal sections of simple shapes, square sticky notes, unit cubes, etc. to demonstrate side lengths are additive. Use hook-and-loop tape if needed for manipulating pieces.
    - Provide grid (graphic or digital) to place shapes on for measurement calculations.
        - Enhance grid with tactile, visual, auditory qualities.
        - Provide manipulatives that can be used to measure/count on the grid.
        - Provide measuring tools with accommodations used in Lesson 1.
3. Provide a worksheet of shapes divided into less complex shapes.
    - Provide complex shapes composed of cut-outs of simple shapes.
        - Provide a model of complex shapes that the student can use a "puzzle" template to construct a complex shape from cut-out sections of simple shapes. Use hook-and-loop tape if necessary.
    - Provide grid (graphic or digital) to place shapes on for measurement calculations.
        - Enhance grid with tactile, visual, auditory qualities.
        - Provide manipulatives that can be used to measure/count on the grid.
        - Provide measuring tools with accommodations used in Lesson 1.

b. Establish Goals/Objectives for the Lesson

Inform the students that in this lesson they may put quadrilaterals or complex shapes together or decompose them. They may also divide quadrilaterals and complex shapes into other shapes to find area using formulas and models.

Explain that they will:

1. Use appropriate known formulas for area.
2. Determine the correct measurement to use.
3. Determine the area of quadrilaterals by composing and decomposing figures.
4. Determine the area of complex shapes by composing and decomposing figures.

Multiple means of representation: In addition to posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journal, computer, graphic organizers (premade or original), etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

Pre-teach concepts of composing and decomposing.

- Use other symbols and terms the student may be familiar with such as "+" and "-"
- Use quadrilaterals and complex shapes cut into simple shapes to illustrate the concepts.
- Make puzzle templates and use hook-and-loop tape.
- Add tactile, visual, and auditory qualities as necessary.

Provide any previous accommodations for measuring and determining area.

Body

Provide students with Lesson 2 Body worksheet.

1. Model how to find the area of the trapezoid.
   a. Work with the students to decompose the figure into a rectangle and triangle and ask the students: What two figures have we made?
   b. Work with the students to determine what dimensions they will need in order to find the area of the rectangle. Label the figure with any missing dimensions.
   c. Find the area of the rectangle by filling in the Area Chart. (A completed chart is shown below for the trapezoid.)
   d. Work with the students to determine what dimensions they will need to find the area of the triangle. Label the figure with any missing dimensions.
   e. Find the area of the triangle by filling in the Area Chart.
   f. Ask the students: We know the area of the rectangle and triangle, how can we now find the area of the trapezoid?

      Figure the area of the trapezoid. See worksheet problem #1.

Figure	Area formula	Area formula with values	Area (unit^2)
Rectangle	A = l x w	A = 4 x 8	32 inches^2
Triangle	A = ½ x b x h	A = ½ x 4 x 4	8 inches^2

Total area = 32 in^2 + 8 in^2 = 40 inches^2

2) Model finding the area of the bedroom from the floor plan by first asking the students: What will be our first step in finding the area of the floor plan?
   a) After selecting a decomposition, ask the students: Are there any dimensions we need to determine before we can find the area?
   b) If there are missing dimensions, have the students determine and label those dimensions on the drawing.
   c) Find the areas of the parts by filling in the Area Chart. (A completed chart is shown below for the bedroom floor plan.)
   d) Find the area of the bedroom.
      i) See worksheet problem #2.

e) To make sure that students realize there is more than one way to decompose the figure and solve the problem correctly, ask the students: Is there any other way to divide the figure to find the area?

Figure	Area formula	Area formula with values	Area (unit^2)
Rectangle	A = l x w	A = 14 x 6	84 feet^2
Rectangle	A = l x w	A = 11 x 6	66 feet^2

Total Area = 84 + 66 = 150 feet^2

3) Model finding the area of the shaded portion of the figure.

a) Have the students complete the Area Chart while answering the questions.

b) Ask the students: What is the area of the entire square, both shaded and non-shaded portions?

c) Ask the students: What is the area of the non-shaded portions?

d) Ask the students: Knowing the areas of both the entire figure and non-shaded portion, what is the best way to find the area of the shaded portion?

e) Ask the students: What is the area of the shaded portion of the figure?

i) See worksheet problem #3.

Figure	Area formula	Area formula with values	Area (unit^2)
Square	A = l x w	A = 20 x 20	400 m^2
Square	A = l x w	A = 9 x 9	81 m^2

Total Area = 400 – 81 = 319 m^2

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area, or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc., to complete the exercises. Present information within the context of student interests (pets, gardening, new bedroom floor plan, etc.)

Additional Considerations for Emerging Readers and Emerging Communicators

1) _____

   a) Divide the trapezoid into a rectangle and triangle that the student can decompose.

      i) Provide digitally or on paper/foam-board with additional tactile and visual qualities.

   b) Provide the definitions and characteristics, graphic organizers, and shapes (paper or digital) from Lesson 1: Introduction for the student to use in identifying the shapes.

      i) Provide formulas, measuring tools, etc. from Lesson 1: Introduction and Body. Use strategies from Lesson 1: Introduction and Body.

      ii) Have the student label missing dimensions by writing, keyboarding, cut and paste prewritten numerals, AT, etc.

   c) Provide chart with accommodations consistent with those provided in Lesson 1: Practice, 2.

   d) Provide formulas, measuring tools, etc. from Lesson 1: Introduction and Body. Use strategies from Lesson 1: Introduction and Body.

      i) Have the student label missing dimensions by writing, keyboarding, cut and paste prewritten numerals, AT, etc.

   e) Provide a chart with accommodations consistent with those provided in Lesson 1: Practice, 2.

   f) Provide graphic (paper or digital with additional tactile, visual, or auditory characteristics as necessary) showing that the rectangle + the triangle = the trapezoid.

      i) Have the student connect the area measurements of the two shapes into the same formula so the student can calculate the area of the trapezoid.

      ii) Use calculator (talking, large button, handheld, or computer-based).

2) _____

   a) Provide paper or digital bedroom floor plan on a scaled grid with additional tactile, visual, or auditory qualities.

i) Show the student two ways it can be decomposed, and allow the student to choose, using preferred mode of communication including AAC/AT, the one with which he/she would like to work.

ii) Look at each dimension individually and mark (sticky note, circle, highlight, etc.) any dimensions that are missing after decomposing.

b) Have the student determine missing dimensions using measuring tools and strategies provided in previous lesson activities.

i) Label the now-solved-for dimensions using previous strategies and tools.

c) Using strategies and tools for completing charts, have the student fill in the Area Chart.

d) Same as c.

e) Provide decomposition options from a. above and have the student demonstrate decomposition not used for Area Chart.

3) _____

a) Follow steps 2.a.-d. above.

e) Using the chart, have the student answer the question using his/her preferred mode of communication.

i) Allow the student to select what he/she believes to be the best way to find the area of the shaded portion and use his/her preferred mode of communication to answer.

ii) Preplan the answer to contribute.

Practice

1. Provide students with Lesson 2 Practice worksheet. Have students work in pairs on a variety of problems, using different complex figures and real-world situations.
2. Give each student an area chart to complete for each problem.
   - Students may use models to explore the various possibilities to complete the charts.
   - Worksheet answer key provides one possible way to find the area, students might determine a different way.
3. Bring the group back together.
   - Ask one student from each pair to discuss the results.
   - After reviewing the results, demonstrate to the students that even if the complex shape was decomposed in different ways, the area was always the same.

Multiple means of representation: Provide students with a copy of the worksheet and the table. Have manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models. Allow students to solve the problem in more than one way.

Multiple means of engagement: Ensure each student is actively involved in his/her pair. Present different problems related to student interests. As you observe pair work, use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Have the student work with a partner who understands the student's mode of communication.
   - The partner might also use that same mode of communication so that he/she serves not only as a math partner but a communication model, too.
     - Make sure that the student participates in all aspects of the activity.
2) Provide models, measuring tools, adapted worksheets using the same accommodations/supports/strategies used up to this point in the lesson.
3) Preplan a contribution to the discussion.
   - Have the student demonstrate different ways that a complex shape might be decomposed, using tactile or digital means.

Closure

a. Revisit/Review Lesson and Objectives

Remind students of the lesson's objectives and expected student outcomes. Have students discuss which of the lesson's problems addressed the objectives below and what they learned from their experiences.

Students will solve real-life and mathematical problems involving area, surface area, and volume.

1. Use appropriate known formulas for area.
   - Ask the students: What are the formulas we have been using to find area?
2. Determine the correct measurement to use.
   - Ask the students: What is the measurement we use when measuring area?

3. Determine the area of quadrilaterals by composing and decomposing figures.
    - Ask the students: How can we find the area of quadrilaterals that are not squares and rectangles?
4. Determine the area of complex shapes by composing and decomposing figures.
    - Ask the students: How do we find the area of complex shapes?

Multiple means of representation: Along with posted lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats: writing, drawing, creative expression, discussion, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide all previous utilized accommodations/supports for using formulas for area.
2) Provide all previous utilized accommodations/supports for determining area.

   Provide answer choices for the student to choose from.
3) Provide all previous utilized accommodations/supports for composing and decomposing complex shapes.
4) Provide all previously utilized accommodations/supports for determining area of complex shapes.

b. Exit Assessment –

1) Have the students solve the following problem:

   The local park is having a design contest for a garden it is going to build. You have decided to enter the design contest. Each design must meet the following conditions: The overall dimensions of the garden are 20 meters by 20 meters. The garden plan must include sidewalks or pathways and green areas, which will not be included in the square footage of the flowerbeds. A fountain may also be included but will not be included in the square footage of the flowerbeds. Indicate any flowerbeds on the garden plan by shading the area(s). The total area of the flowerbeds must be at least 150 square meters but

cannot exceed 200 square meters. Have fun and be creative designing our park's new garden.

2) Have the students individually design/create a garden with beds that are complex shapes.
    - The garden and beds need to have the dimensions indicated.
    - The flowerbeds should be shaded.
    - The total area of the garden, with flowerbeds, must be at least 150 square meters but cannot exceed 200 square meters.
    - Provide the students with paper/pencil, graph paper, ruler, and area chart.

Multiple means of representation: Allow the students to have a written copy of the problem, area formulas, and area table. Have drawings and manipulatives available for students to use. Allow students to refer back to their work samples, models, drawings, notes, etc.

Multiple means of expression: Students may draw or use manipulatives or use the computer to model solutions.

Multiple means of engagement: Create situations that include areas of interests to the students.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide the word problem with symbol-based text, tactile symbols, pictures, etc.; provide a text reader or read the problem to the student.
    - Simplify the wording; take out non-relevant information.
    - Highlight the important information.
2) Provide all accommodated materials and strategies including definitions, examples, formulas, measuring tools, charts, etc.
    - Provide several simple and complex shapes on grid paper (tactile/visual qualities added) or digitally (auditory qualities added) that the student can choose from for the flowerbeds. The student must calculate the area of each shape and determine which ones he/she can use to stay within the range given.
        - Provide cues to decomposing the complex shapes.
        - Simplify by using complex shapes that decompose into squares and rectangles only.
        - Indicate dimensions of shapes.
    - Secure selected flowerbeds with glue, tape, or hook-and-loop tape.

Mathematics Geometry Unit
Lesson #3

Objectives

- Students will solve real-life and mathematical problems involving area, surface area, and volume.

Essential Questions

- How do we find the surface area of a rectangular prism?
- Where is surface area used in real life, and why is it important?

Vocabulary

Area	Rectangle
Edge	Rectangular prism
Face	Surface area
Height	Vertices
Length	Width
Net	

Materials

Area formula chart including surface area formula

Graph paper

Introduction to an activity table

Ruler

Scissors

Sticky notes

Tape

Various rectangular prisms (e.g., cereal boxes, tissue boxes)

Worksheet

- Lesson 3 Practice
- Lesson 3 Exit Assessment

Introduction

a. Activate Previous Knowledge

1) Review with students how they have been finding the area of two-dimensional figures.
   a. Ask students – Is a box a two-dimensional figure?
   b. Lead a discussion about how two-dimensional figures and three-dimensional figures differ.
   c. Have students locate two-dimensional figures and three-dimensional figures in the classroom.
2) Give each pair of students the net of a rectangular prism and some tape.
   a. Have the students explore/problem solve how to construct a rectangular prism.
   b. Once the rectangular prisms are constructed, discuss what a face is, what an edge is, and what vertices are.
   c. Have the students complete a table that includes the following categories for their rectangular prisms.

Shape	# of faces	# of vertices	# or edges

   d. After tables are filled out, discuss what the rectangular prisms have in common.
   e. Ask students - Will different rectangular prisms have a different number of faces, vertices, or edges?

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or model at their desks.

Multiple means of expression: Provide a list of definitions to use in completing the table or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc.

Additional Considerations for Emerging Readers and Emerging Communicators

1)_____

a. Provide a small box and a cutout or drawn figure (outlined in glue, etc., colored, or textured as necessary) the same size as the length and width of the box.

b. Provide concrete representations of several common shapes (rectangle/square, sphere, cylinder, pyramid, etc.) and cutout or drawn figures (outlined in glue, etc., colored, or textured as necessary) that correspond in exact size and shape.

   Provide digital representations of 2-D and 3-D shapes that the student can manipulate on the screen (using AT). Add sound for interest.

c. Have the student use the shapes (2-D and 3-D) provided in 1. b. to find and categorize (by 2-D or 3-D, not by shape) figures in the environment.

   - Record shapes by taking digital photos (using AT).

2) _____

a. Provide net of rectangular prism with each face

   - numbered
   - colored differently
   - textured differently

   Provide a 3-D shape exactly corresponding to the net so the student has a more solid surface to construct the net around.

b. Have the student put a colored dot on each face (numbered sequentially from 1-6 if necessary).

   - Have the student use a marker to highlight each edge.
   - Use a different color marker or dots marked with "V"s to indicate the vertices.

   Have the student put a numbered face icon on each face.

   - outline each edge with glue, yarn, etc.
   - indicate each vertex with a colored or textured dot or sticker.

c. Provide table with symbol-based text.

   Provide table with visual/tactile cues used on the actual net.

   - put face icon on the face column of the table
   - put yarn on the edge column

- put textured dot on the vertices column

Have the student fill out table after counting each characteristic (using the "+1" function on a calculator if necessary) by:

- writing the numbers;
- cutting and pasting pre-printed numbers;
- verbally stating the number, which the partner scribes;
- selecting the answer from two to several options) completing the table digitally (using AT), etc.

d. Let the student choose characteristics of rectangular prisms on the table to answer the question (using his /her preferred mode of communication).

e. Provide nets (adapted the same way as the first net was above) of several other, differently sized rectangular prisms that the student can use to complete similarly adapted tables so he/she can develop his/her answer.

b. Establish Goals/objectives for the Lesson

Inform students that they will solve real-life and mathematical problems involving area, surface area, and volume.

Explain that they will:

1. Describe the characteristics of a rectangular prism.
2. Construct the net of a rectangular prism.
3. Determine the surface area of a rectangular prism from a net.
4. Determine the surface area of a rectangular prism using a formula.
5. Describe when surface area is used and why it is important in real life.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journal, computer, graphic organizers (premade or original), etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide rectangular prisms and tables the student used in the first part of the Introduction.
2) Provide a net of a rectangular prism.
3) same as 2.
4) Provide a formula using symbols and textures.
5) Provide examples of situations where determining surface area is important to "me".
    - Examples may be captioned picture representations of situations
    - Examples may be digital animations of situations
    - Examples may be tactile representations with auditory captions

Body

Form student groups and provide each group with a rectangular prism/box of the same size. Tell the students that they will be turning the rectangular prism/box into a net like the paper nets they just folded in the previous activity. Students may work as a whole group with the teacher or in small groups. Note: There is more than one net for a rectangular prism.

1) Make a sketch of the rectangular prism/box.
    a. Ask the students: For the rectangular prism/box, what are the length, width, and height?
    b. Measure and record each of these dimensions.
2) Label the faces of the rectangular prism/box before flattening it to make a net.
    a. Use sticky notes labeled: Front Face, Back Face, Left Side Face, Right Side Face, Top Face, and Bottom Face.
    b. Work together to label the rectangular prism's/box's faces.
    c. Explain that by labeling the faces, it will be easier to visualize how the net makes up the rectangular prism/box.
3) Make the net.
    a. Cut the rectangular prism/box so that each face of the rectangular prism/box is still connected.
    b. Brainstorm with the students about where to cut before making cuts.

c. Once the cuts are made, show the students the flattened rectangular prism/box which is now a net.

d. Have the students draw the net on paper or graph paper.

e. Ask the students: Can you see how the net drawing could be cut out and taped together to form the rectangular prism/box?

f. Have the students include the fold lines on the sketch.

g. Have the students label the faces on the net sketch.

h. Measure the lengths and widths of the rectangles of the flattened rectangular prism/box and have the students record these on the net sketch.

    i. See pages in Lesson 3 Resources.

4) Discuss with students how they have been finding the area of two-dimensional figures.

Ask the students: Since the net is a two-dimensional figure, what is its area?

Students may use an area chart to find the area.

Shape/Face	**Area Formula**	**Formula with values**	**Area (unit^2)**

Total Area =

5) Explain to the students that when a figure is three-dimensional, surface area is found.

Ask the students: How would the surface area of the rectangular prism/box compare to the area of the net?

6) Instruct students that there is a formula for finding the surface area of a rectangular prism: Surface Area = 2 (lw + hl + hw).
    a. Ask the students: From the measurements made on the rectangular prism/box, what is the surface area using the formula?
    b. From the net, find the area of each face using the chart.
        i. See pages in Lesson 3 Resources

Prism Surface	**Face Area**
Top Face	l x w
Bottom Face	l x w
Front Face	h x l
Back Face	h x l
Left Side Face	h x w
Right Side Face	h x w

7) If we know the area of each face, how can we find the surface area of the figure? (Add together all of the faces.)
8) How does the surface area found by the formula compare to the surface area found by adding together the areas of the faces? (They are the same.)
9) Since they are the same, let's see how the formula was developed.
    a. First, horizontally list the faces and put the area formula below.
    b. Top + Bottom + Front + Back + Left + Right = Surface area
    c. lw + lw + hl + hl + hw + hw = Surface area
10) Use algebra to simplify:
    a. 2lw + 2hl + 2hw = Surface area (Combine like terms.)
    b. 2(lw + hl + hw) = Surface area (Factor out a 2.)

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or model at their desks. Provide a list of area formulas.

Multiple means of expression: Allow students to solve problems using formulas and/or models and record information into tables using various formats (paper and pencil, computer, etc.) Allow students to use a reference of formulas.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, models, computer, etc.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Allow the student to work directly from the rectangular prism rather than the sketch.
    a. Mark the length, width, and height directly on the prism.
    b. Allow the student to use the same supports for measuring and documenting measurements as provided in Lesson 1- Body.
2) Add extra visual (symbols or color) or tactile cues (texture, etc.) to the sticky note labels.
3) Outline where the cuts should be made.

    Allow the student to use adaptive or electric scissors.

    Have the student trace the flattened prism/net onto graph paper graph paper enlarged to 1-inch squares.

    Have the student label the faces on the net using the same supports used in labeling the faces of the prism.
4) Have the student determine the area of each face, by measuring and using the formula or by counting using a grid and/or manipulatives, and record in the table. Allow the student to use whatever supports he/she has been using for measuring, calculating, and recording.
5) – 6) Allow the student to use either formula or manipulatives to find area of the entire net (surface area of rectangular prism).
    a. Have the student compare the net to the folded out prism by laying one on top of the other.
        - Have the student compare the net to the rectangular prism by reassembling the prism and then folding the net around it. Hook-and-loop tape could be used instead of sticky tape. (Pre-teach or review the concept of "same" if necessary using systematic instruction techniques.)
    b. Allow the student to use a calculator (handheld, digital, big button, talking, etc.) or manipulatives.

- Show the two calculations and ask if they are the same. (Pre-teach or review the concept of "same" if necessary using systematic instruction techniques.)

7) – 8) Provide choices in the student's preferred communication mode.

9) Model how the top and bottom are represented by length and width, two sides are represented by height and width, and two sides are represented by height and length.

10) Provide a template for the formula.

   a. Allow the student to use a calculator.

   b. Student may use manipulatives instead of the formula.

Practice

1. Have the students work in pairs on a variety of problems using different rectangular prisms and nets.
   a. Give each student Lesson 3 Practice worksheet.
   b. Once the groups have completed the problems on the worksheet, ask one student from each pair to discuss the results.
2. Demonstrate to students that there is more than one possible way to create a net for a rectangular prism, but the surface area remains the same.

Multiple means of representation: Provide students with a copy of the word problem, a template for the formulas for area and a table to find composed areas. Have drawings and manipulatives available for the students to use.

Multiple means of expression: Provide a list for formulas to determine area or provide options for using manipulatives and/or computer models, etc. Students may draw or use manipulatives or use the computer to model solutions.

Multiple means of engagement: Ensure that each student is actively involved in his/her pair. Allow students to use paper/pencil, manipulatives, computer, etc.

Additional Considerations for Emerging Readers and Emerging Communicators

If the student is struggling with the concept of surface area after the completing the introduction and body of this lesson, it might be helpful to provide additional instruction using the ***Lesson 3 Concept Reinforcement Activity (CRA)*** *prior to prior to this practice activity of Lesson 3.*

1. Provide all supports used in Lessons 1-3 up to this point including those for measuring, calculating, and recording.
    a. Provide worksheet with supports used in Lesson 3 – Introduction.
        i. Put formula with correctly measured numbers on each face.
        ii. Provide rectangular prisms and nets representing the problems.
        iii. Provide digital representations and access for manipulation using AT.
    b. Allow the student to use all supports and materials provided to contribute to discussion.
    c. Allow the student to use his/her preferred mode of communication.
    d. Preplan one contribution to the discussion.
2. Provide one rectangular prism with several possible nets that can be folded around the prism to illustrate the concept.

Closure

d. Revisit/Review Lesson and Objectives

Remind students of the lesson's objectives and expected student outcomes. Have students discuss which of the lesson's problems addressed the objectives below and what they learned from their experiences.

Students will solve real life and mathematical problems involving area, surface area, and volume.

1) Describe the characteristics of a rectangular prism.
2) Construct the net of a rectangular prism.
3) Determine the surface area of a rectangular prism from a net.
4) Determine the surface area of a rectangular prism using a formula.
5) Describe when surface area is used and why it is important in real life.

Ask students: How are area and surface area connected? Brainstorm with students when they might use area and surface area in everyday life.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats through writing, drawing, creative expression, discussion, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me".

Additional Considerations for Emerging Readers and Emerging Communicators

1. As review is conducted, provide the student with all previously used materials, strategies, and supports.
2. Preplan information for the student to contribute to the discussion using his/her preferred mode of communication.

Exit Assessment

Give the students a new word problem to solve that includes finding surface area.

Have students work independently to find the surface areas of the two boxes and solve for the situation. Students should justify their answer.

- See Lesson 3 Exit Assessment Worksheet for drawings of the wooden boxes.

Above are two sample wooden boxes they have for sale in a shop. You want to buy one to cover with carpet to make your cat a scratching post. However, you only have 40 square feet of carpet left over from having new carpet put in the house. Do you have enough carpet to cover either box or only one? Support your decision by showing your work.

Multiple means of representation: Provide students with a copy of the word problem and a template of the formulas for area. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow the students to solve the problem using formulas and/or models.

Multiple means of engagement: Create situations that include areas of interests to the students.

Additional Considerations for Emerging Readers and Emerging Communicators

Provide all supports previously used in Lessons 1-3.

- Summarize the word problem.
- Highlight important information in the word problem.
- Provide a number line representing square feet with the acceptable range of square feet of carpet highlighted.

As this is now assessment as opposed to all previous activities in this lesson, the student's first independent attempts solving the two problems and answering the question should be observed and recorded for accuracy.

Mathematics Geometry Unit
Lesson #4

Objectives

- Students will solve real-life and mathematical problems involving area, surface area, and volume.

Essential Questions

- How do we find the volume of a rectangular prism?
- Where is volume used in real life, and why is it important?
- What is the difference between surface area and volume?

Vocabulary

Cubic Centimeter	Rectangular Prism
Cubic Foot	Surface Area
Cubic Inch	Volume
Height	Width
Length	

Materials

Cubes

- Centimeter cube
- Inch cubes
- Foot cubes

Box

Grid paper

Rubik's® cube

- Graphic representation of Rubik's® cube

Ruler

Scissors

Template of the formula for volume

Water bottle

Worksheet - Lesson 4 Practice

Introduction

a) Activate Previous Knowledge

1. Remind students that they have been working on the concepts of area and surface area.
    - Review with students the units used to measure area and surface area (square centimeters, square inches, square feet, and square meters).
    - Discuss how they can use this knowledge to solve problems they encounter in the "real world".
2. Present the students with a Rubik's® cube and a box.
    - How would you find the surface area of each?
    - How many Rubik's® cubes will fit in the given box?
    - How can we find out how many Rubik's® cubes will fit in the given box?

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or model at their desks.

Multiple means of expression: Provide a list of formulas to determine surface area or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc.

Additional Considerations for Emerging Readers and Emerging Communicators

If the student has had little to no experience with the concept of volume or the skill of calculating volume, it might be helpful to provide instruction using the ***Lesson 4 Concept Reinforcement Activity (CRA)*** *prior to teaching the Introduction to Lesson 4.*

1. _____
    - Provide the student with all materials and supports he/she has previously used in Lessons 1-3, including definitions, examples, formulas, etc.
    - Provide examples in symbol-based text, with added visual and tactile characteristics, photos, digital animations with sound accessible through AT, etc.
2. Allow the student to use all materials and supports to answer questions using his/her preferred mode of communication.

- Provide options for the student to select from.

3. Allow the student to move the Rubik's® cube around in the box before estimating.
   - Provide options for the student to select from.

b. Establish Goals/Objectives for the Lesson

Inform students of the goals/objectives for this lesson. Share with them that they will make decisions as to the appropriate measurements and formulas to use in solving real world and mathematical problems involving the volume of rectangular prisms.

Explain that they will:

1. Describe and understand what volume measures and the correct units for volume measurements.
2. Determine the volume of rectangular prisms using manipulatives and formulas.
3. Describe when volume is used and why it is important in real life.
4. Describe and understand how surface area and volume of solids differ.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journal, computer, graphic organizers (premade or original), etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide definition and unit measures for volume with or without symbol-based text, with textures, other tactile qualities, colors, etc. (resources such as Standard Tactile Symbol List available from the Texas School for the Blind www.tsbvi.edu/tactile-symbols may be useful in determining representations; some representations may need to be pre-taught (Rowland, 2012), but use of the same representations/symbols [or the system] during this and other lessons will reduce the need for pre-teaching).
2. Provide definitions and images of rectangular prisms with or without symbol-based text, with textures, other tactile qualities, colors, etc.

- Provide formulas and examples of volume of rectangular prisms with or without symbol-based text with textures, other tactile qualities, colors, etc.

3. Provide calculator (large button, handheld, talking, or computer-based).
4. Provide manipulatives and concrete representation at least one rectangular prism to use in calculating volume. Add textures, color, sound, etc. as needed.
5. Provide real-life examples of using volume with or without symbol-based text.
   - Provide real-life examples of using volume with pictures.
   - Provide real-life examples of using volume with animated digital displays including sound.
6. Provide graphic organizer, such as a T- chart showing the differences between surface area and volume.
   - Add symbol-based text to organizer and information.
   - Put information on cards so the student can sort information into correct columns.
   - Use hook-and-loop tape.
   - Add texture to outline graphic organizer and background of column categories.
   - Add color for interest.
   - Color code information according to categories so the student can match colors to sort information.
7. Provide graphic organizer digitally with sound so the student can manipulate information using AT.

Body

1. Lead a discussion about what volume is: the measurement of space occupied in three dimensions.
   - Show students that volume is the space taken up inside of something.
   - Ask the students: When we put the Rubik's® cubes in the box, what were we finding?
   - Give an example of a cubic unit such as a one-inch cube. Explain to students that since volume is a measurement made in three dimensions, the units used are cubic.
   - Also show students a cubic centimeter and cubic foot.
   - Ask students: What unit would you use when measuring the volume of a cereal box?

2. Find the volume of a box using inch cube blocks and the formula.
    a. Have students see how many inch cube blocks it takes to fill a box.
    b. Ask the students: If the blocks fill all of the space in the box, what is the volume of the box?
    c. Ask the students:
        - What is the length of the box?
        - What is the width of the box?
        - What is the height of the box?
    d. Show the students that the volume of the box can be found by multiplying length times width times height.
        - Ask the students: How does the volume found using cube inch blocks compare to using the formula?
            - Ask the students: What would be the volume of the pictured box?
3. Discuss with students how surface area and volume measurements differ.
    a. Given the scenario of Sophia using wrapping paper to wrap a gift in a box, ask the students: What part relates to surface area? (*the wrapping paper, because it covers the entire surface of the box*)
    b. What part relates to volume? (*the gift, because it must fit inside the box*)
    c. What unit would you use to measure the wrapping paper?
    d. What unit would you use to measure the volume?

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desk.

Multiple means of expression: Provide the formula to determine the volume of a rectangular prism, or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc., to complete exercises. Present information in context of student interests (pets, gardening, new bedroom floor plan, etc.).

Additional Considerations for Emerging Readers and Emerging Communicators

1. _____

    a. Using the rectangular prism and manipulatives provided in the introduction, have the student mirror the teacher's demonstration of volume.

    b. Allow the student to use his/her preferred mode of communication.

    c. Give the student a one-inch cube.

    d. no additional supports

    e. Give the student concrete representations of a cubic centimeter and a cubic foot.

    f. Give the student a cereal box and have him/her explore which will fit inside – the inch cube or the foot cube.

    g. Provide the representations digitally, and provide access through AT.

    h. Allow the student to use his/her preferred mode of communication to answer.

2. _____

    a. Give the student enough inch cubes to fill the cereal box.

    b. Provide the cubes and box digitally with access through AT.

    c. Cut the front from the box on three sides, leaving the bottom connected so that the front can be "hinged" or folded back. This will allow the student to fill the box while the box lay on its back, creating a more visually easy way to see when the box is full.

    d. Count the cubes that fill the box.

        - Touch and count.
        - Place the cubes on a number line.
        - Use a calculator with the "+1" function.

    e. Allow the student to use his/her preferred mode of communication.

    f. Indicate where to find the length, width, and height of the box on the sides of the box (on digital access, too, if that is being used).

    Have the student measure (use any supports previously used in measuring) or line up the inch cubes to find the dimensions of the box (this may be digital, as well).

    g. Provide the formula.

        - Allow the student to use a calculator.

- Provide choices of "same" or "different" so the student can choose the answer using AAC.

3. Provide representations of the gift and wrapping paper (rectangular prism, net, photos, digital, etc.).
    a. Provide the student with the choices of "gift" and "paper" in his/her preferred mode of communication. Allow the student to select the answer. Correct as necessary demonstrating why the answer(s) is correct.
    b. Provide the choices of "inches" and "square inches" in the student's mode of communication. Allow the student to select the answer. Correct as necessary demonstrating why the answer(s) is correct.
    c. Provide the choices of "inches" and "cubic inches" in the student's mode of communication. Allow the student to select the answer. Correct as necessary demonstrating why the answer(s) is correct.

Practice

1. Have students work in pairs on a variety of problems using different rectangular prisms with some students only using dimensions to determine volume. Use Lesson 4 Practice Worksheet.
2. Give each student the formula for finding the volume of a rectangular prism: $V = l \cdot w \cdot h$
3. Bring the whole group back together.
    a. Have a student from each pair discuss how they found the answer to a question.
    b. After going over the practice, ask students: Is important to know how much an object holds?
    c. Is it important to know the measure of the surface? Why or why not?

Multiple means of representation: Provide students with a copy of the word problems and a template of the volume formula. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problems using formulas and/or models.

Multiple means of engagement: Ensure each student is actively involved in his/her partnership. Present different problems related to student interests. As you observe group work, use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide inch cube manipulatives, which can represent different cubic units of measure in the problems (inch, centimeter, foot, meter, etc.) and measurement, calculation, and writing supports previously provided (including, digital, AAC, and AT).
2. Provide the formula using the same supports as previously provided.
3. _____?

    a. Preplan a discussion contribution using the student's preferred mode of communication.

    b. Allow the student to use previously provided examples to answer the question.

    c. Allow the student to use previously provided examples to answer the question.

Closure

b. Revisit/Review Lesson and Objectives

Remind students of the lesson's objectives and expected student outcomes. Have students discuss which of the lesson's problems addressed the objectives below and what they learned from their experiences.

Student will solve real-life and mathematical problems involving area, surface area, and volume.

1. Describe and understand what volume measures and the correct units for volume measurement.
2. Determine the volume of rectangular prisms using manipulatives and formulas.
3. Describe when volume is used and why it is important in real life.
4. Describe and understand how surface area and volume of solids differ.

Ask students: What are some examples of situations in which we use volume in real life?

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats through writing, drawing, creative expression, discussion, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. As review is conducted, provide the student with all previously used materials, strategies, examples, and supports.
2. Preplan information for the student to contribute to the discussion using his/her preferred mode of communication.

Exit Assessment

Give students a new problem:

Students will be making models of an open cardboard box to find the greatest volume they can create from a piece of cardboard measuring 10 cm by 12 cm. The dimensions of the box are to the nearest whole centimeter. Students will use a piece of grid paper to make the model.

1. Step one: Cut out a 10-unit by 12-unit square from the grid paper.
2. Step two: Cut a 1 by 1 square out of each corner and fold to form a box.
3. Step three: Record the length, width, and height of the box in the chart and figure the volume.
4. Step four: Cut 2-unit-by-2-unit squares from each corner and fold to form a box.
    - Repeat steps two and three.
5. Step five: Cut 3-unit-by-3-unit squares from each corner and fold to form a box.
    - Repeat steps two and three.
6. Step six: Cut 4-unit-by-4-unit squares from each corner and fold to form a box.
    - Repeat steps two and three.

Use the table to answer this question: What are the dimensions of the box with the greatest volume?

Length of side corner	Length of box	Width of box	Height of box	Volume of box
1 cm				
2 cm				
3 cm				
4 cm				

Multiple means of representation: Provide students with a copy of the word problem, drawn models of the situation, and/or formulas as needed/requested.

Multiple means of expression: Allow students to solve the problem using the formula, drawings, computer graphics, and/or models.

Multiple means of engagement: Create situations that include areas of interests to the students.

Additional Considerations for Emerging Readers and Emerging Communicators

1) Provide all supports previously provided (measuring, calculating, definitions, formulas, worksheets, examples, writing, cutting, etc.).
2) Provide visual/tactile/verbal cues for folding.
3) Precut and pre-fold the paper into boxes.
4) Provide digital representations of the boxes.
5) Decrease the number of problems to solve.
6) Provide answer selections.

As this is now assessment as opposed to all previous activities in this lesson, the student's first independent attempts solving the problems and answering the question should be observed and recorded for accuracy.

Mathematics Geometry Unit

Lesson 5: Culminating Activity

Objectives:

Students will:

- Determine the volume of rectangular prisms.
- Determine the surface area of rectangular prisms.
- Identify and quantify attributes of a problem that need to be measured.
- Draw a net of a rectangular prism.

Essential Questions:

1. What is the volume of a rectangular prism?
2. What is the surface area of a rectangular prism?
3. How do volume and surface area differ?
4. What is the net of a rectangular prism?

Vocabulary

Area	Rectangular prism
Complex shape	Square inch
Cubic inch	Square unit
Cubic unit	Surface area
Height	Volume
Length	Width

Materials

Culminating Activity Resource Part A

Culminating Activity Resource Part B

Drawings

Grid paper

Tables

Introduction

a. Revisit/Review Unit and Lesson Objectives

Remind the students that throughout these lessons they were to solve real-life and mathematical problems involving area, surface area, and volume.

1. Describe and understand what area measures and the correct units for area measure.
2. Determine the area of rectangles, squares, and triangles using graph paper/manipulatives and formulas.
3. Use appropriate known formulas for area.
4. Determine the area of complex shapes by composing and decomposing figures.
5. Construct the net of a rectangular prism.
6. Determine the surface area of a rectangular prism from a net or using a formula.
7. Describe when surface area is used and why it is important in real life.
8. Describe and understand what volume measures and the correct units for volume.
9. Determine the volume of rectangular prisms using manipulatives and formulas.
10. Describe and understand how surface area and volume of solids differ.
11. Describe when volume is used and why it is important in real life.

Conduct a class discussion on which skills were used to solve different types of problems. Talk about additional strategies they used to implement the skills and solve the problems.

Multiple means of representation: Along with posted lesson objectives in the classroom, students may refer to their individual copies of the objectives and their mathematics journals.

Multiple means of expression: Students can share what they have learned or strategies they used by showing different models, pictures, drawings, etc., used throughout the lessons.

Multiple means of engagement: Share ideas of how these skills have been useful in solving the problems

Additional Considerations for Emerging Readers and Emerging Communicators

As review is conducted, provide the student with all previously used materials, strategies, examples, and supports from previous lessons. Utilize the strategies were the most helpful.

Preplan information for the student to contribute to the discussion using his/her preferred mode of communication.

Exit Assessment – Culminating Activity

Scenario:

Mrs. Sweet owns Grandma's Candied Apples. She makes candied apples and sells them to small shops and large stores. Decisions must be made as to the best boxes for packaging multiple candied apples to be sent to small shops and large stores.

Three designs have been presented to her for packaging her candied apples to sell to small shops. For small shops, each candied apple is in its own package, which measures 6 inches by 6 inches by 10 inches.

For large stores, Mrs. Sweet needs to have a box designed and net drawn of that box for manufacturing. The candied apples will be packaged in threes. The candied apples box now measures 6 inches by 18 inches by 10 inches. Each large shipping box needs to hold 18 apples.

To keep costs down, Mrs. Sweet knows that the boxes she selects must not have more volume than needed. Also, the box with the least surface area will cost less in materials to produce.

Tasks: Part A

Select the best shipping box for small stores.

1. Determine how many candied apples will fit into Box A, Box B, and Box C.
    - See Culminating Activity Resource Part A Worksheet for drawings of boxes and chart.
2. Determine the volumes of Box A, Box B, and Box C.
3. Determine the surface area of Box A, Box B, and Box C.
4. Make a chart of the above measurements.

	Box A	Box B	Box C
How many apples does the box hold?	12	12	12
Volume of the box	4320 in^3	4320 in^3	4752 in^3
Surface area of the box	2424 in^2	864 in^2	1392 in^2

5. Taking into account Mrs. Sweet wants a box without extra volume and the least amount of surface area, which box best fits Mrs. Sweet's needs? Why?

Tasks: Part B

Design the best shipping box for large stores.

1. Design a shipping box that will hold 18 apples packaged in groups of 3.
2. Determine how many packages each shipping box will need to hold.(i.e. 18/3 = 6 packages of 3 apples each)
3. Make a chart showing the box dimension possibilities as well as the volume and surface area for each.

Dimensions - length, width, height	Volume	Surface Area
108 x 6 x 10	6480 in^3	3576 in^2
54 x 12 x 10	6480 in^3	2616 in^2
36 x 18 x 10	6480 in^3	2376 in^2
54 x 6 x 20	6480 in^3	3048 in^2
27 x 12 x 20	6480 in^3	2208 in^2
18 x 18 x 20	6480 in^3	2088 in^2
18 x 12 x 30	6480 in^3	2232 2

4. Taking into account Mrs. Sweet wants a box without extra volume and the least amount of surface area, which box from your designs best fits Mrs. Sweet's needs?
5. Draw a net of your box design that best fits Mrs. Sweet's needs.

Multiple means of representation: Allow students to refer to their mathematics journals and other notes as they solve the problem. Provide students with a copy of the word problem and the tables. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models and record information into the tables using various formats (computer, paper/pencil, drawings, etc.)

Multiple means of engagement: Ensure each student is actively involved in solving the problem. Encourage students to consider options for solving the problem that will engage them. As you observe students work, use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

Tasks: Part A

As this is now assessment as opposed to all previous activities in this lesson, the student's first independent attempts solving the problems and answering the question should be observed and recorded for accuracy. If correct answers are required to answer subsequent questions, make a note if the student is not correct, supply the correct information, and move to the next part of the task. For example, if the student does not calculate volume correctly for one box, supply the correct answer so he/she has accurate information to answer the question about which box best fits Mrs. Sweet's needs.

- Provide the student with all previously used materials, strategies, examples, and supports including labeled representations of the boxes.
- Reduce the number of boxes.
- Provide the word problem in symbol-based text.
- Summarize the word problem.
- Highlight important information in the word problem.
- Read the problem to the student or have the student use text reader.
- Give the student step-by step instructions one at a time.
- Provide answer choices for the student to select from.
- Add formulas into chart.

Tasks: Part B

1)Provide access to all the supports used in Tasks: Part A.

2)Decrease the complexity of the task by providing the student with the dimensions of two boxes on the chart instead of requiring the student to create his/her own boxes. Add formulas, etc. to the chart and have the student determine volume and surface area of each (again, providing all supports used in Part A.

3)Provide pre-drawn/cut nets (one for each of the boxes) and have the student identify or select which net matches the box they chose in step 6.

4)As this is now assessment as opposed to all previous activities in this lesson, the student's first independent attempts solving the problems and answering the question should be observed and recorded for accuracy. If correct answers are required to answer subsequent questions, make a note if the student is not correct, supply the correct information, and move to the next part of the task. For example, if the student does not calculate volume correctly for one box, supply the correct answer so he/she has accurate information to answer the question about which box best fits Mrs. Sweet's needs.