National Center and State Collaborative

Functions Content Module

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for post-secondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

IDEAs that Work
U.S. Office of Special Education Programs

The contents of this resource were developed as part of the National Center and State Collaborative for a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

These materials and documents were developed under the National Center and State Collaborative (NCSC) General Supervision Enhancement Grant and are consistent with its goals and foundations. Any changes to these materials are to be consistent with their intended purpose and use as defined by NCSC.

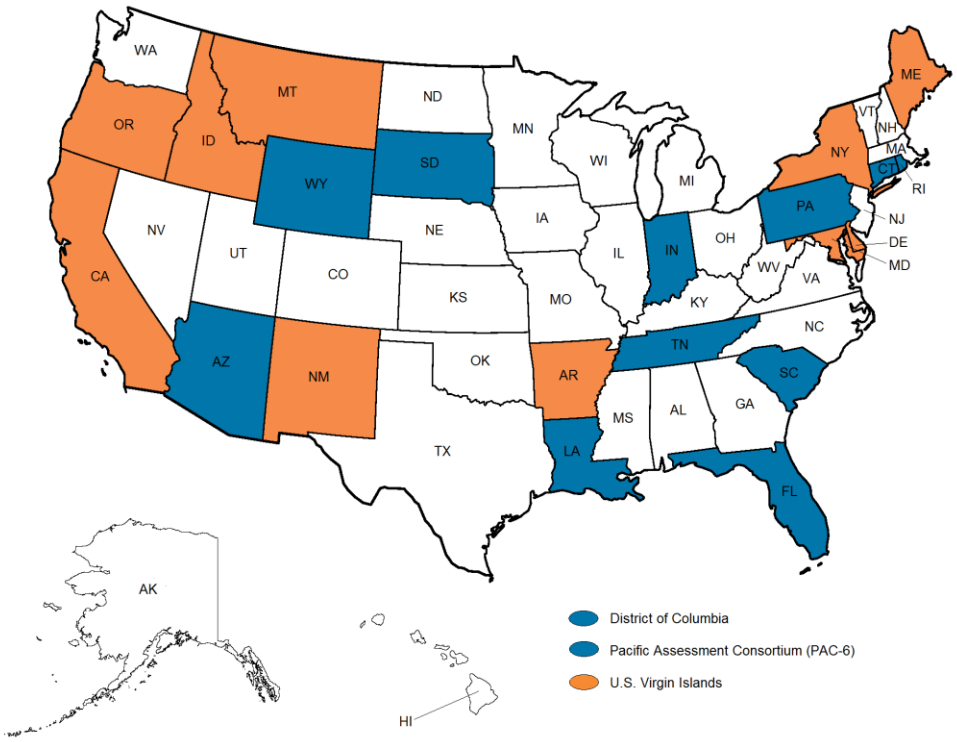
This document is available in alternative formats upon request.

National Center and State Collaborative

NCSC is a collaborative of 13 states and five organizations.

The states include (shown in blue on map): Arizona, Connecticut, District of Columbia, Florida, Indiana, Louisiana, Pacific Assessment Consortium (PAC-6)[1], Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, Montana, New Mexico, New York, Oregon, and U.S. Virgin Islands.



*Core partner states are blue in color and Tier II states are orange in color.

[1] The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

National Center and State Collaborative

The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.

150 Pillsbury Drive SE
207 Pattee Hall
Minneapolis, MN 55455
Phone: 612-708-6960
Fax: 612-624-0879
www.ncscpartners.org

National Center and State Collaborative

Functions Content Module

July 2014

Table of Contents

Plot the Course

http://www.worthwhilesmile.com/air-balloons-kaleidoscope/

The Rationale

Remember the old saying “a picture is worth a thousand words?” The ability to look a graph and interpret its meaning is an integral skill in both everyday life and almost any profession. For example, when you receive your electricity bill, a graph is often included so that the customer can see what months they used the most electricity at a quick glance.

Module Goal

The goal of this module is to provide detailed instruction on the more difficult concepts of functions (i.e., linear, and quadratic) to teachers of students with disabilities at the middle and high school level. This module promotes a mathematical understanding of these concepts so that a teacher can begin to plan how to teach the concepts to students. Additionally, this module will provide instructors with potential adaptations and modifications to consider when designing materials and instruction for students with severe disabilities.

Module Objectives

After viewing the content module, teachers will:

1. Identify independent and dependent variables
2. Identify a linear function
3. Identify a quadratic function
4. Discriminate between functions and non-functions when provided graphs

Time for Take Off

Understanding the vocabulary used within functions is important for both teachers and students in planning and implementing math lessons. As a teacher, knowing and using the mathematical terms not only ensures your instruction stays true to the math content, but will also help with collaborating with other math teachers or content experts. When choosing which vocabulary to teach, it is most important that the teacher selects the most salient, important, or most frequently used vocabulary for each lesson.

Below you will find a list of vocabulary included within this module. It may or may not be necessary to provide instruction for all terms as students may have learned them previously. Functions are mostly covered in middle school so vocabulary for this content module has been combined. If you are a high school teacher and are not confident your students know some of these vocabulary terms, you may want to review and teach some unknown terms in the focus and review part of your lesson plan.

While providing vocabulary instruction, you may consider including pictures or objects to make the instruction more concrete for students with disabilities (See Ideas to support vocabulary learning below).

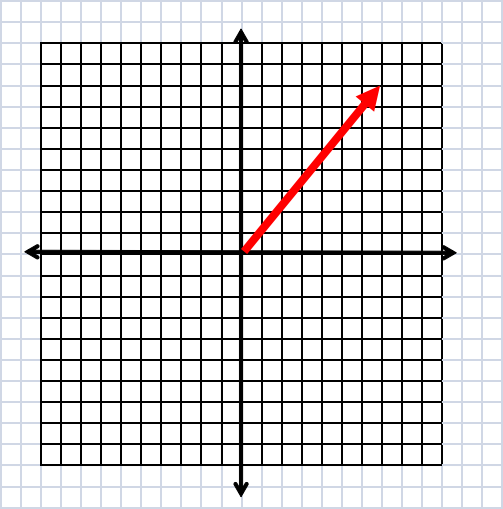
Vocabulary

- Variable – a term that includes a letter that represents a value (e.g., in the equation $y = 2x + 4$, 2x is the variable).
- Constant – a fixed value (e.g., in the equation $y = 2x + 4$, 4 is the constant)
- Independent variable – what is being manipulated in the situation. For example, when measuring a child's height as they get older, the aging process is the independent variable (age is what is being changed).
- Dependent variable – what is changed as a result of manipulating the independent variable. For example, when measuring a child's height as they get older, their height is the dependent variable (it changes as a result of the independent variable).
- Function – mathematical operations demonstrating the relationship between the input and output of an expression.
- Linear function – represented by an equation in the form of $y = mx + b$; when graphed the coordinates provided will form a straight line.
- Quadratic function – represented by the an equation in the form of $y = mx^2 + bx + c$; when graphed the coordinates provided will form a U-shape.

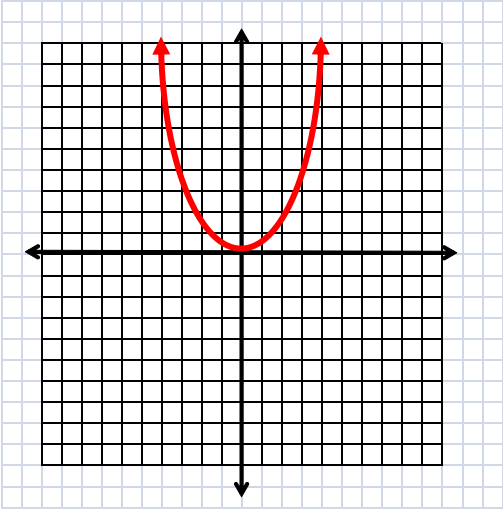
- Exponential function – represented by an equation in the form of $y = mx$, where a is a positive number not 1; a common exponential function is the growth of some organism (including humans).

Ideas to Support Vocabulary Learning

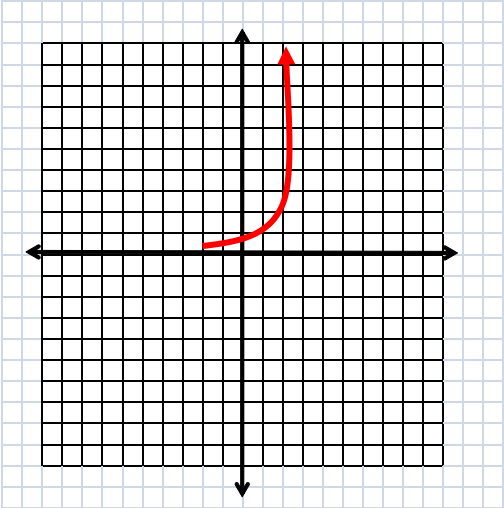
- Teach students to discriminate between linear, quadratic, and exponential functions based on the appearance of graphs.



Linear function



Quadratic function



Exponential function

Floating on Air

Before you can begin teaching functions, you need a deep understanding of these mathematical concepts. Some of these concepts may be familiar to you. Below is a list of skills that should be covered at each grade level. For concepts that you need more information about, please view the accompanying PowerPoint presentations that will walk you through an example as well as make some suggestions for instruction.

Middle and High School

In middle and high school skills include:

- 6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation
- 8.PRF.2c1 Given two graphs, describe the function as linear and not linear
- 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables
- H.PRF.2c1 Make predictions based on a given model (for example, a weather model, data for athletes over years)

Insert Linear Functions PowerPoint here

Insert Exponential Functions PowerPoint presentation here

Insert Quadratic Functions PowerPoint presentation here

Great! Now that you have viewed the PowerPoint presentation most useful to you, the next section will provide some ideas to consider when planning for Universal Design for Learning.

NOTE: All PowerPoint presentations begin with the same “What is a function” slides for review.

Linear Functions

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

- A function is a special mathematical operation demonstrating the relationship between the **input** (also called domain) and **output** (also called range) of an expression
  - For example,
    - The relationship between number of gallons of gas purchased and the total cost of the purchase (see the chart below)

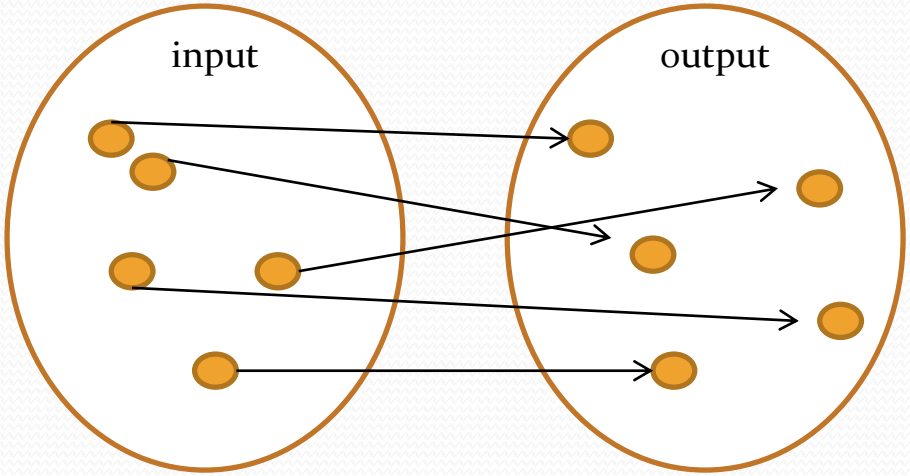
Note: the relationship is constant

Input: Number of gallons	**Relationship:** $3.00 per gallon	**Output:** Total purchase
1	X $3.00	$3.00
2	X $3.00	$6.00
3	X $3.00	$9.00
4	X $3.00	$12.00

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

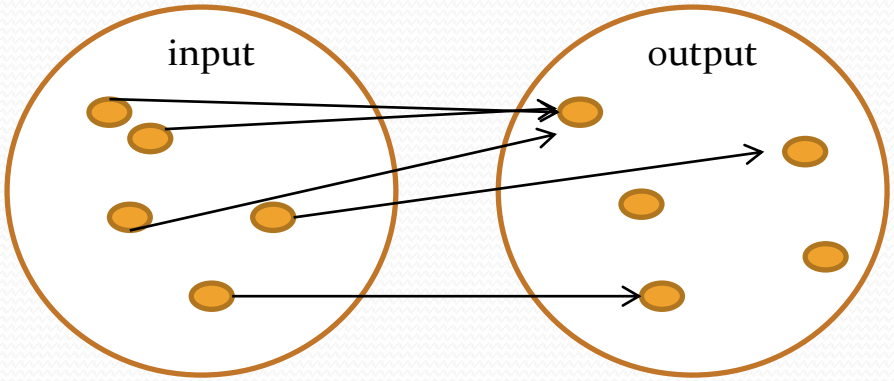
- Unlike other mathematical relationships, in **functions**
  - The relationship must be one, consistent regardless of the input value
  - Function must be true for every possible input value



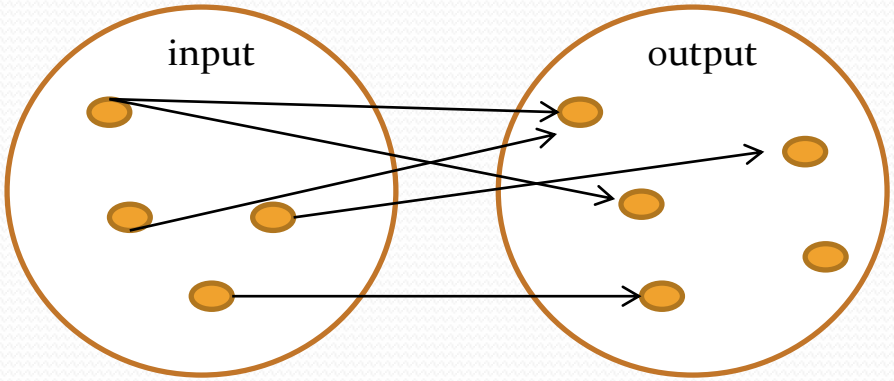
This is a function because there is only one output for each input

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?



This is a function because there is only one arrow coming from each of the input

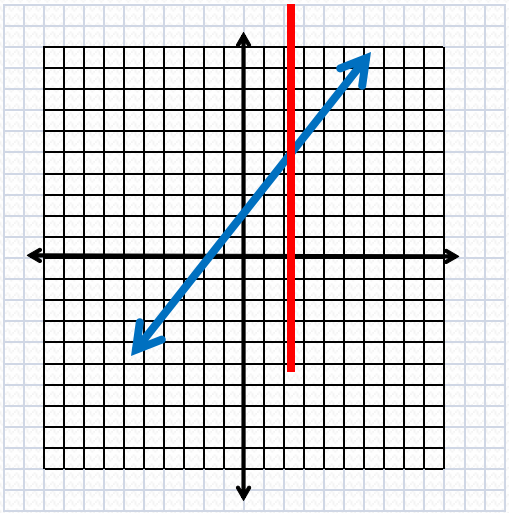


This is a **NOT** function because there is more than one arrow coming from one of the input values

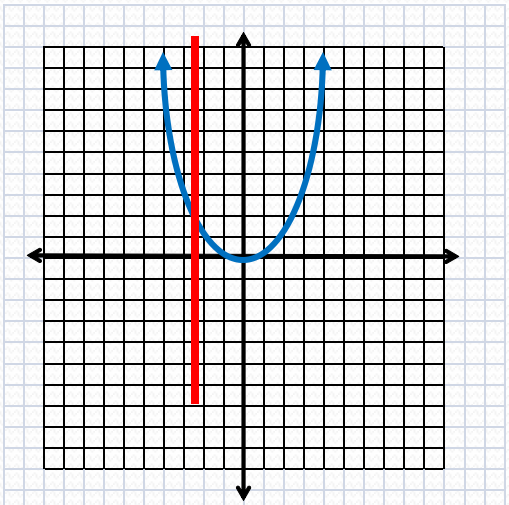
The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

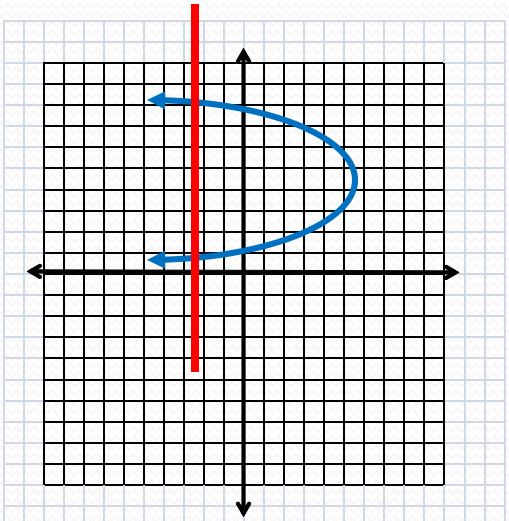
- Another method to tell if something is a function is called the "vertical line test"



Function: the vertical line only crosses one point



Function: the vertical line only crosses one point



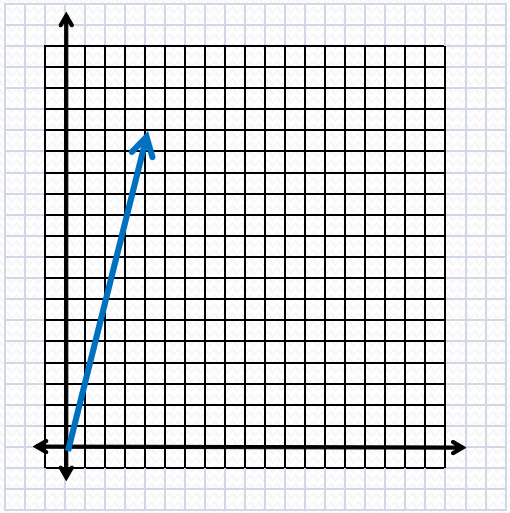
NOT a function: the vertical line crosses more than one point

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a linear function?

- Any function that when the input and output are graphed, the input/output pairs form a line
  - Using the gas example from the previous slide

Input: Number of gallons	**Relationship:** $3.00 per gallon	**Output:** Total purchase
1	X $3.00	$3.00
2	X $3.00	$6.00
3	X $3.00	$9.00
4	X $3.00	$12.00



Note: The input the x-coordinate and the output is the y-coordinate

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Is the function linear?

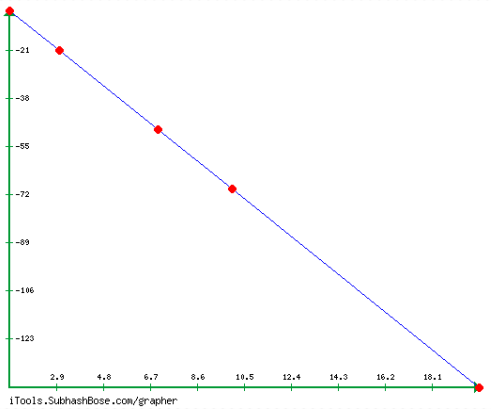
Input (x-coord)	Relationship	Output (y-coord)
1		-7
3		-21
7		-49
10		-70
20		-140

Method 1: is the relationship consistent?

Input (x-coord)	Relationship	Output (y-coord)
1	X -7	-7
3	X-7	-21
7	X-7	-49
10	X-7	-70
20	X-7	-140

Answer: Yes, the relationship is consistent

Method 2: graph the coordinates



Answer: Yes, the graphed coordinates are linear

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Linear functions: An example

- The standard from for linear equations is

$$Y=ax+b$$

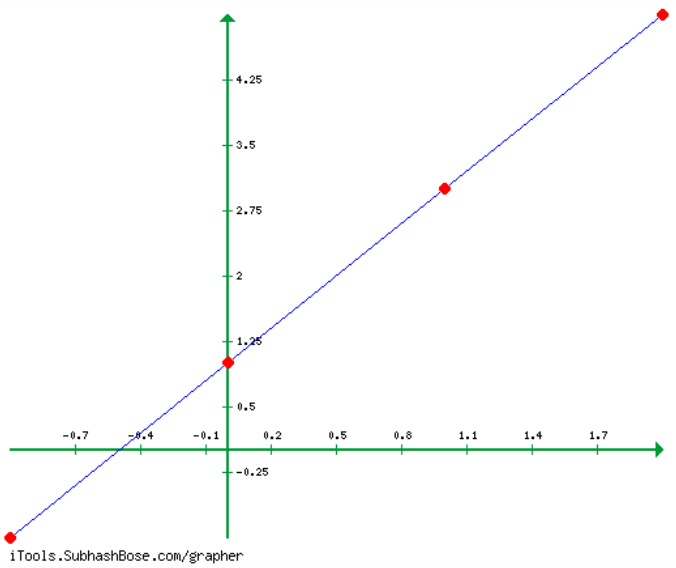
- Problem: Graph the linear function $y=2x+1$
  - Step One: create a table with input and determine output values

Input	Output
2	2(2)+1= **5**
1	2(1) +1= **3**
0	2(0)+1= **1**
-1	2(-1)+1= **-1**

The input and output values are the ordered pairs to graph

Linear functions: An example

- Step 2: Graph the ordered pairs



Is the graph linear?

YES, THIS IS A LINEAR FUNCTION

Is the function linear: Graphing calculators

- In most classrooms, students may not graphs functions by hand (especially in high school). Therefore, it may be beneficial to create a task-anlaysis for entering these equations.
  - May also want to consider color-coding buttons and terms in the equation to assist student with limited numeracy skills

Ideas for application

- Create personally-relevant word problems or contexts
  - The amount you pay for gas and the number of gallons
  - How much electricity you use and the total of your monthly bill

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Making connections

- Linear functions with exponents addresses the middle and high school Core Content Connectors of
  - 6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation
  - 8.PRF.2c Given two graphs, describe the function as linear and not linear
  - 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Exponential Functions

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

- A function is a special mathematical operation demonstrating the relationship between the **input** (also called domain) and **output** (also called range) of an expression
  - For example,
    - The relationship between number of gallons of gas purchased and the total cost of the purchase (see the chart below)

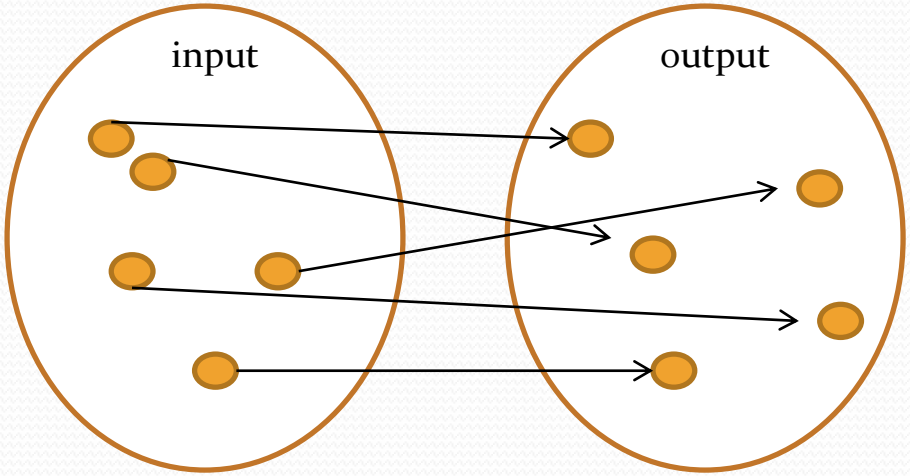
Note: the relationship is constant

Input: Number of gallons	**Relationship:** $3.00 per gallon	**Output:** Total purchase
1	X $3.00	$3.00
2	X $3.00	$6.00
3	X $3.00	$9.00
4	X $3.00	$12.00

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

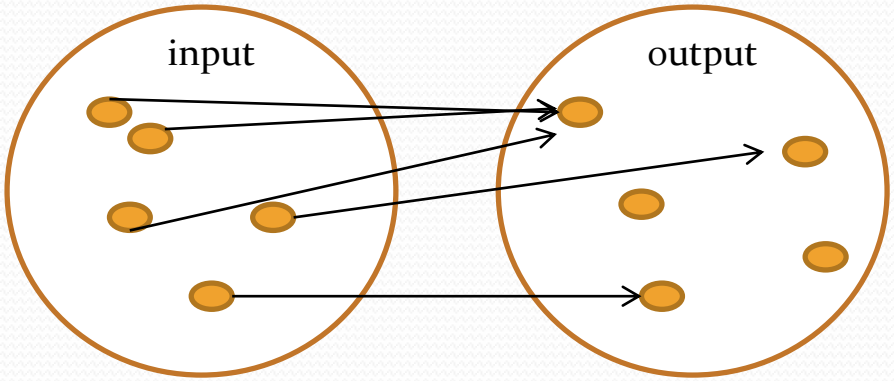
- Unlike other mathematical relationships, in **functions**
  - The relationship must be one, consistent regardless of the input value
  - Function must be true for every possible input value



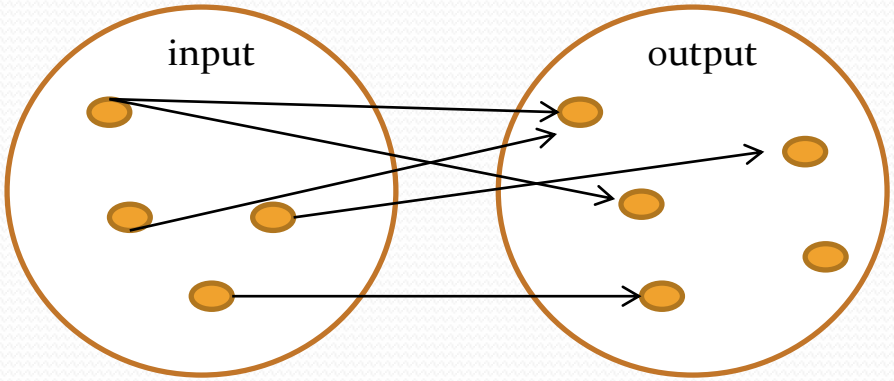
This is a function because there is only one output for each input

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?



This is a function because there is only one arrow coming from each of the input

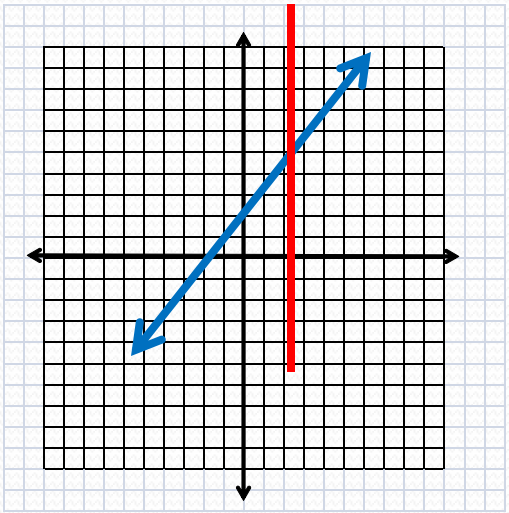


This is a **NOT** function because there is more than one arrow coming from one of the input values

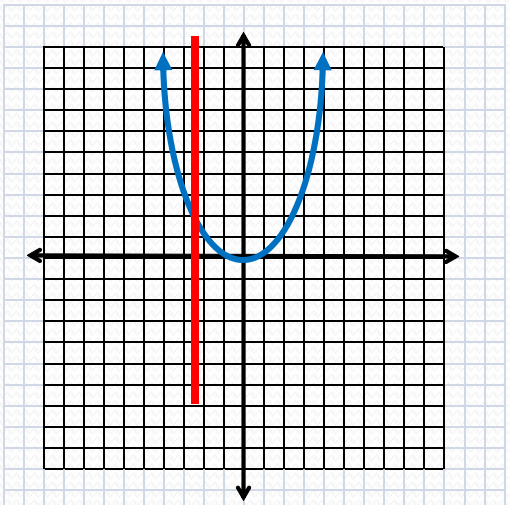
The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

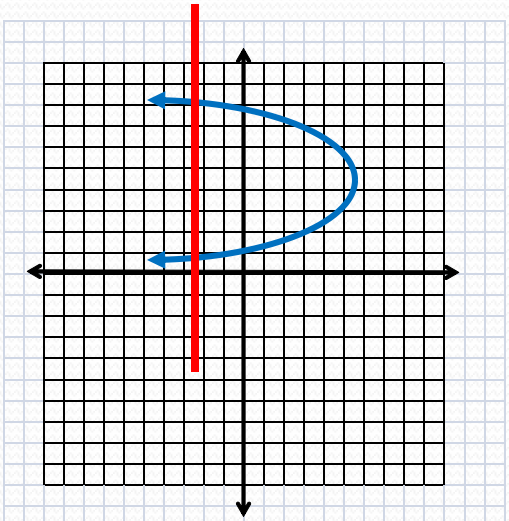
- Another method to tell if something is a function is called the "vertical line test"



Function: the vertical line only crosses one point



Function: the vertical line only crosses one point



NOT a function: the vertical line crosses more than one point

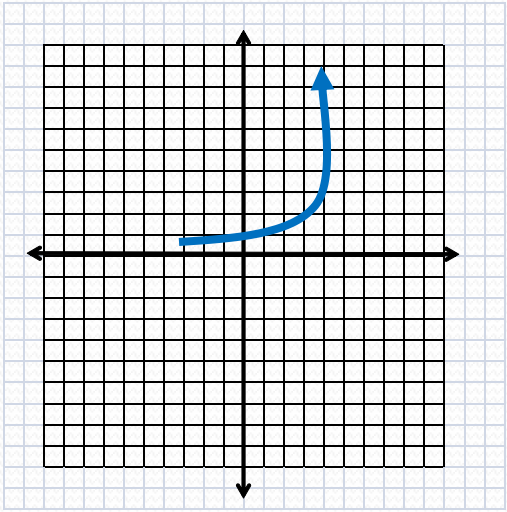
The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is an exponential function?

- A function that when the input and output are graphed, the input/output pairs form a "growth" shape

- Typically written in the form $y=a^x$
  - A is a positive number that is not 1

NOTE: unlike other functions, the variable is the exponent



The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Is the function exponential?

- There are many methods to verify whether or not a function is exponential; however, the remaining slides will only focus on ONE method…..graphing
  - Specifically, we will discuss graphing by hand and using a graphing calculator

- Remember, this is NOT THE ONLY WAY! For information on the other methods, please see a mathematics education professional

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

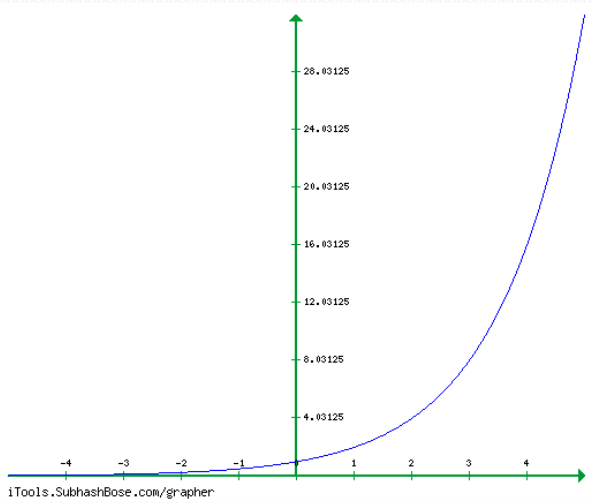
Is the function exponential: Graph it by hand

$$f(x) = 2^x$$

Step 1: Create a table with input and solve to find output values?

Input (x-coord)	Output (y-coord)
2	$2^2 = 4$
1	$2^1 = 2$
0	$2^0 = 0$
-1	$2^{-1} = 0.5$
-2	$2^{-2} = 0.25$

Step 2: graph the coordinates



Answer: Yes, the graphed coordinates form a "growth"shape with a vertex at (-1,1)

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Is the function exponential: Graphing calculators

- In most classrooms, students may not graphs functions by hand (especially in high school). Therefore, it may be beneficial to create a task-anlaysis for entering these equations.
  - May also want to consider color-coding buttons and terms in the equation to assist student with limited numeracy skills

Ideas for application

- Create personally-relevant word problems or contexts

- Link to other curricular areas such as science
  - Bacteria growth and how it relates to spreading of germs or handwashing
- Solve using graphing calculators
  - The skills used for entering equations into a graphing calculator could be generalized to data entry on a computer or using a cash register in a retail setting

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Making connections

- Exponential functions with exponents addresses the middle and high school Core Content Connectors of
  - 6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation
  - 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables
  - H.PRF.2c1 make predictions based on a given model (for example, a weather model, data for athletes over years)

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Quadratic Functions

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

- A function is a special mathematical operation demonstrating the relationship between the **input** (also called domain) and **output** (also called range) of an expression
  - For example,
    - The relationship between number of gallons of gas purchased and the total cost of the purchase (see the chart below)

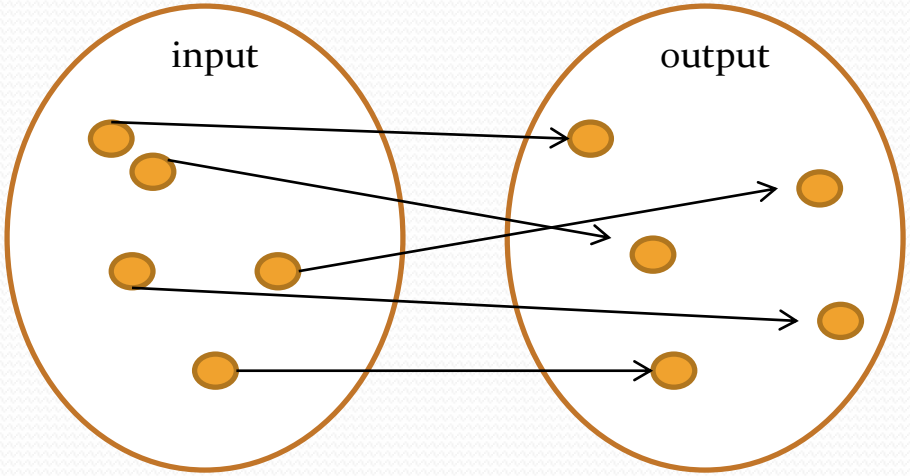
Note: the relationship is constant

Input: Number of gallons	**Relationship:** $3.00 per gallon	**Output:** Total purchase
1	X $3.00	$3.00
2	X $3.00	$6.00
3	X $3.00	$9.00
4	X $3.00	$12.00

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

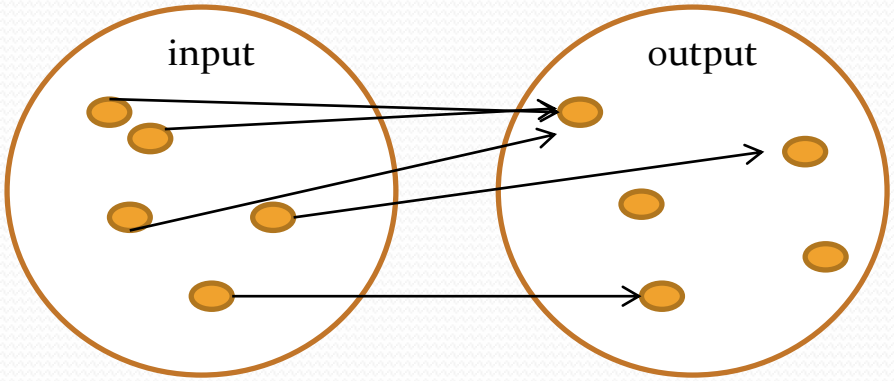
- Unlike other mathematical relationships, in **functions**
  - The relationship must be one, consistent regardless of the input value
  - Function must be true for every possible input value



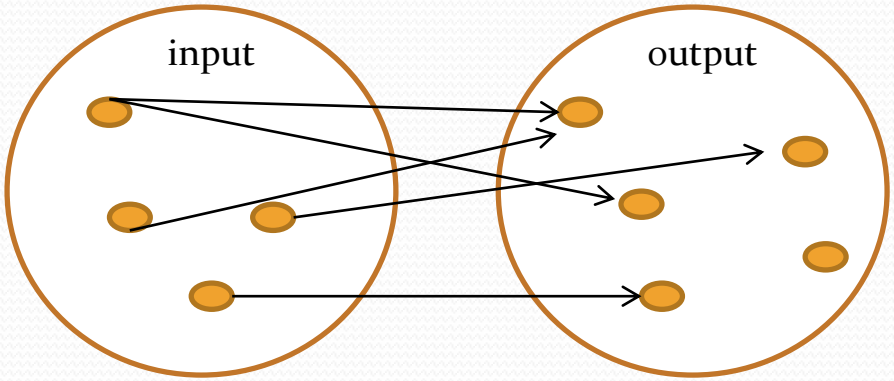
This is a function because there is only one output for each input

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?



This is a function because there is only one arrow coming from each of the input

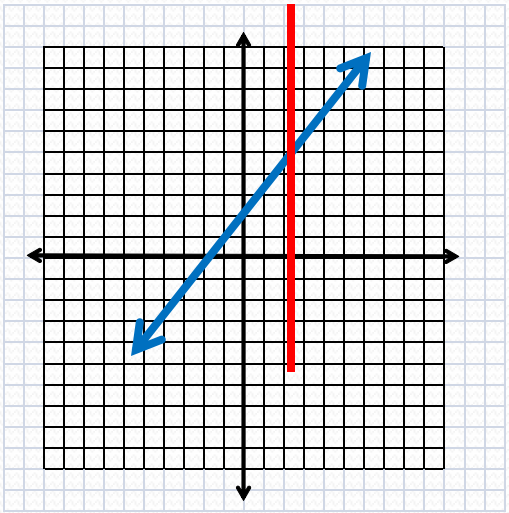


This is a **NOT** function because there is more than one arrow coming from one of the input values

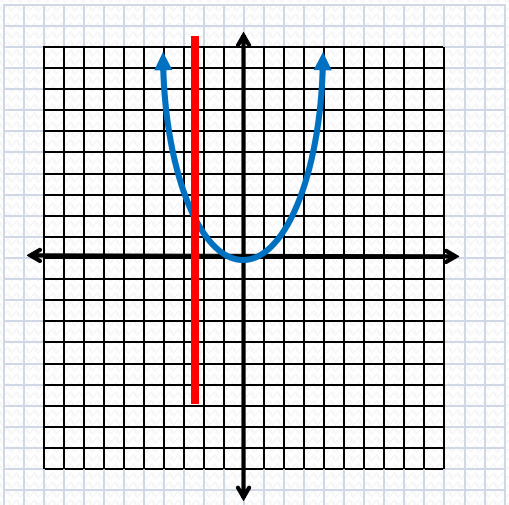
The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a function?

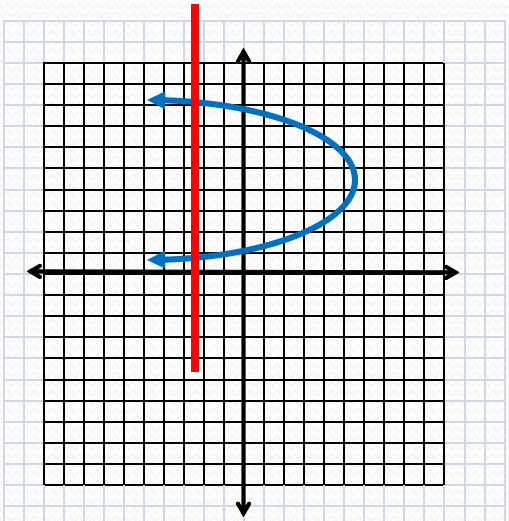
- Another method to tell if something is a function is called the "vertical line test"



Function: the vertical line only crosses one point



Function: the vertical line only crosses one point



NOT a function: the vertical line crosses more than one point

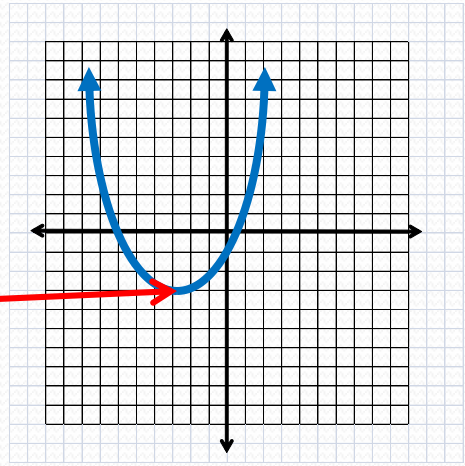
The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

What is a quadratic function?

- Any function that when the input and output are graphed, the input/output pairs form a U-shape called a parabola

- Typically written in the form $y=ax^2+bx+c$
  - a, b, and c are real number and
  - a does not equal o

This point is called the vertex



The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Is the function quadratic?

- There are many methods to verify whether or not a function is quadratic; however, the remaining slides will only focus on ONE method…..graphing
  - Specifically, we will discuss graphing by hand and using a graphing calculator

- Remember, this is NOT THE ONLY WAY! For information on the other methods, please see a mathematics education professional

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

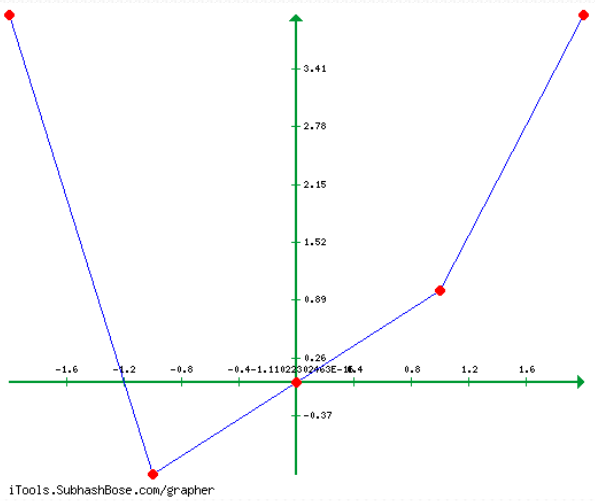
Is the function quadratic: Graph it by hand

$$f(x) = x^2$$

Step 1: Create a table with input and solve to find output values?

Input (x-coord)	Output (y-coord)
2	$(2)^2 = 4$
1	$(1)^2 = 1$
0	$(0)^2 = 0$
-1	$(-1)^2 = 1$
-2	$(-2)^2 = 4$

Step 2: graph the coordinates



Answer: Yes, the graphed coordinates form a U-shape with a vertex at (-1,1)

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Is the function quadratic: Graphing calculators

- In most classrooms, students may not graphs quadratic functions by hand (especially in high school). Therefore, it may be beneficial to create a task-analysis for entering these equations.
  - May also want to consider color-coding buttons and terms in the equation to assist student with limited numeracy skills

Ideas for application

- Create personally-relevant word problems or contexts

- Solve using graphing calculators
  - The skills used for entering equations into a graphing calculator could be generalized to data entry on a computer or using a cash register in a retail setting

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Making connections

- Linear functions with exponents addresses the middle and high school Core Content Connectors of
  - 6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation
  - 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables
  - H.PRF.2c1 make predictions based on a given model (for example, a weather model, data for athletes over years)

The contents of this content module were developed by special educator Bethany Smith, PhD and validated by content expert Drew Polly, PhD at University of North Carolina at Charlotte under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made

Sharing the Sky

UNIVERSAL DESIGN FOR LEARNING

Some examples of options for teaching functions to students who may present instructional challenges due to:				
	Visual Impairment or Deaf/Blind	**Physical Impairment: Little/No Hand Use**	**Lacks Basic Numeracy Concepts**	**Motivational/Attention Issues**
Representation	Add texture to graphs and axis so students can feel the shape of the function's input and output.	Manipulate graphing software using assistive technology (switches, laser pointer).	Always pair equation with its graphic representation.	Enter ordered pairs into graphing software instead of drawing by hand.
Expression	Students scan textured graphs to discriminate between linear, exponential, and quadratic functions; Student states answer or scans raised numbers to select correct answer; use voice output devices for student to select the correct answer.	Focus on students ability to discriminate between functions based on the graph versus computation; use a switch to indicate correct answers; use an eye gaze board to select answer; phrase questions so that they require a "yes/no" response, these can easily be answered using an eye gaze, head turn, two switches, etc.	Enter equations into graphing calculators; use graphing functions to find x and y intercepts or specific ordered pairs; Focus on students ability to discriminate between functions based on the graph versus computation.	Utilize all the technological possibilities when studying functions (e.g., graphing calculators, talking calculators, free online computer software).
Engagement	Create personally-relevant stories to accompany the graph (e.g., for an exponential function develop a story about their own growth from birth to school age).	Use bright colors to call attention to salient feature of the graph (e.g., the shape, where it cross an axis); pair student with another student without a physical impairment and have them work together to create fraction and decimal representations.	Use a graphing calculator or free online graphing software that allows student to enter an equation as it appear and creates the graph for the student.	Utilize graphing software on computer or tablet PCs; pair with another student to complete problems in pairs.

Prepare for Landing

Below you will find ideas for linking functions to real-world applications, the college and career readiness skills addressed by teaching these concepts, module assessments for teachers, sample general education lesson plans incorporating universal design for learning framework, blog for teachers to share their ideas, and a place to upload and share lesson plans from teachers who completed this module.

One way to help assist in a special educator's development within this curricular area is through collaboration with other teachers in your building. Some activities with real world connection include:

- Anything that grows (e.g., bacteria, plants, students)
- Focus on using the mathematic tools (e.g., graphing calculators), these skills can be possibly generalized into data entry or operating a cash register

In addition to the real-world applications of these concepts, skills taught within this content module also promote the following college and career readiness skills.

Communicative competence:
Students will increase their vocabulary to include concepts related to "growth", "output", "relation" or "input" In addition, they will be learning concepts such as "exponent".

Fluency in reading, writing, and math
Students will have an opportunity to increase their numeracy and sight word fluency while participating in problem solving related to "functions" such as number recognition, counting, and interpreting a graph.

Age appropriate social skills
Students will engage in peer groups to solve problems related to "functions" that will provide practice on increasing reciprocal communication and age appropriate social interactions.

Independent work behaviors
By working with real life problems related to "functions" students will improve work behaviors that could lead to employment such as marketing or any job that has to analyze sales rates, stock clerks, order fillers, retail cashier, and laboratory assistant based professions. When providing opportunities for real life problems leave some materials out and prompt/teach the students to determine who they should ask and what they should ask for to be able to solve the problem.

Skills in accessing support systems
At times, students will need to ask for assistance to complete activities related to "functions" which will give them practice in accessing supports. Students will gain practice asking for tools such as graphing calculators or other manipulatives. They can ask a peer to complete the physical movements of the tasks they are not about to do themselves. Be sure to teach students to ask versus having items or supports automatically given to them.

In addition to collaborating with other educational professionals in your building, the following list of resources may also help provide special educators with ideas for activities or support a more thorough understanding of the mathematical concepts presented in this content module.

Additional Resources

- http://itools.subhashbose.com/grapher/ - This website allows you to simply input x and y-coordinates and it will graph the function for you
- http://www.ncpublicschools.org/acre/standards/common-core-tools/ - This website provides an "unpacking document" for the Mathematics Common Core Standards that helps teachers identify what is most important and the essential skills for each standard.
- http://www.shodor.org/interactivate/lessons/ - Website include fully developed and interactive general education lesson plans for a variety of math topics including functions.
- http://www.mathforum.com/ - Website specifically for teachers which provides a variety of ideas and activities to use in your classroom.
- www.teachertube.com - Youtube for teachers! Simply search for your content area and this websites provides a variety of videos including videos of math experts working through math problems step by step (**free registration required).**
- www.ksde.org - This website provides a webinar about how to adapt materials for students who have visual impairments.

Module Assessments

Insert assessment here

Sample General Education lesson plans

Insert developed lesson plans here

Have an idea: Upload the lesson plans you've created here

Insert link for teachers to upload lesson plans

Teacher's Corner: Blog with other teachers

Insert forum or blog for teachers to share ideas

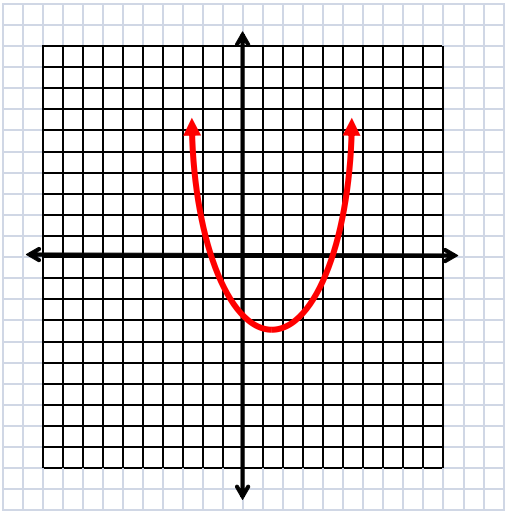
Adapt the following general education lesson plan; adapt, and upload. These lesson plans may be shared with higher education professionals developing strategies to provide meaningful academic instruction in mathematics to students with severe disabilities.

Insert blank lesson plan form with UDL chart here

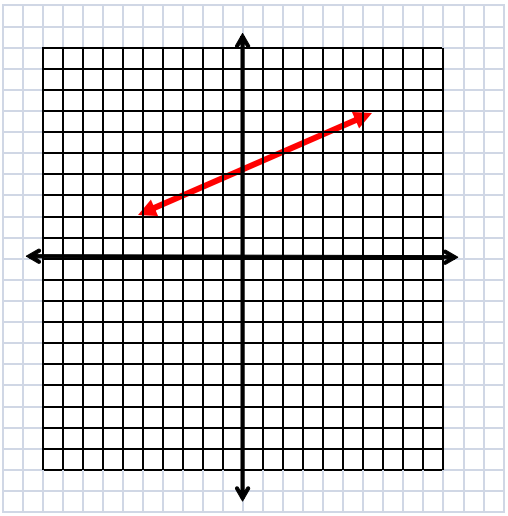
Insert link for teachers to upload lesson plans

Simplifying Expressions Assessment

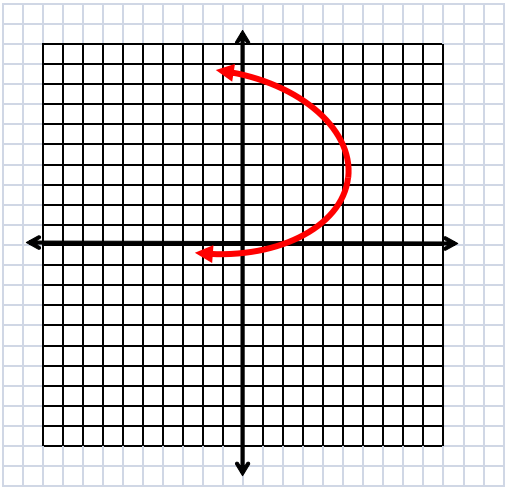
1. What type of function does the graph show?
   a) Linear
   b) Exponential
   c) Quadratic
   d) None of the above



2. What type of function does the graph show?
   a. Linear
   b. Exponential
   c. Quadratic
   d. None of the above



3. What type of function does the graph show?
   a. Linear
   b. Exponential
   c. Quadratic
   d. None of the above

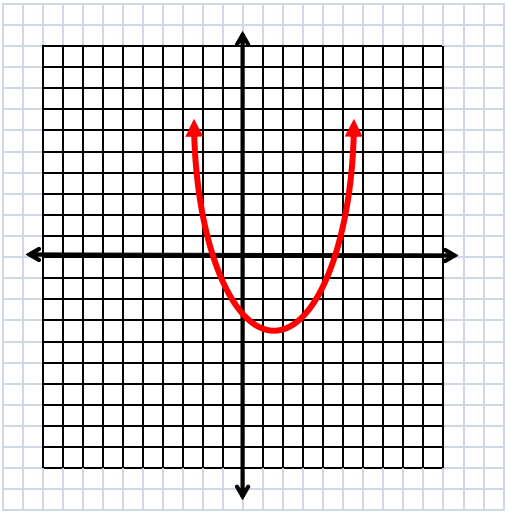


4. What are two methods for determining if an equation is a function?
   a. Create a table, determine output for various input, and graph
   b. Vertical line test
   c. Find the slope
   d. A and B

5. What is the standard formula for an exponential function?
   a. $f(x) = ax + b$
   b. $f(x) = 2^x$
   c. $f(x) = x^2$
   d. None of the above

Simplifying Expressions Assessment: Answer Key

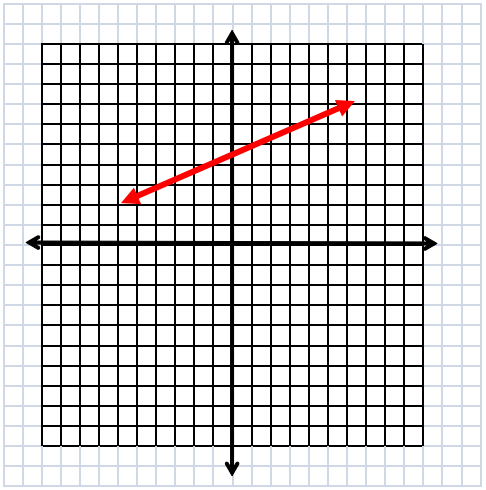
1. What type of function does the graph show?
   a. Linear
   b. Exponential
   c. Quadratic
   d. None of the above



Correct feedback: Yes, the answer is quadratic.

Incorrect feedback: Sorry, the answer is quadratic. Please review the quadratic functions PowerPoint.

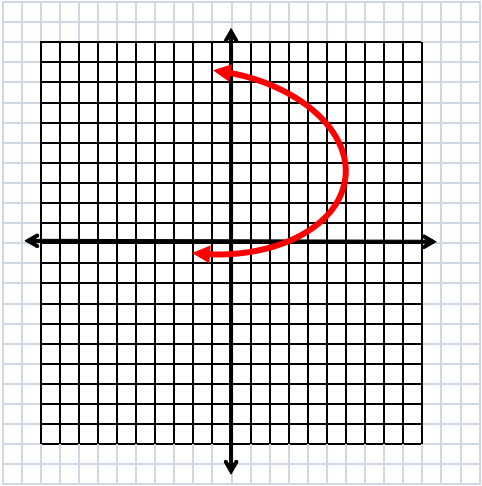
2. What type of function does the graph show?
   a. Linear
   b. Exponential
   c. Quadratic
   d. None of the above



Correct feedback: Yes, the answer is linear.

Incorrect feedback: Sorry, the answer is linear. Please review the linear functions PowerPoint presentation.

3. What type of function does the graph show?
   a. Linear
   b. Exponential
   c. Quadratic
   d. None of the above



Correct feedback: Yes, the answer is none of the above.

Incorrect feedback: Sorry, the answer is none of the above. Please review the What is a Function slides on any PowerPoint in this module.

4. What are two methods for determining if an equation is a function?
   a. Create a table, determine output for various input, and graph
   b. Vertical line test
   c. Find the slope
   d. A and B

Correct feedback: Yes, the answer is A and B.

Incorrect feedback: Sorry, the answer is A and B. Please review the What is a Function slides on any of the PowerPoint presentations.

5. What is the standard formula for an exponential function?
a. $f(x) = ax + b$
b. $f(x) = 2^x$
c. $f(x) = x^2$
d. None of the above

Correct feedback: Yes, the answer is $f(x) = 2^x$

Incorrect feedback: Sorry, the answer is $f(x) = 2^x$. Please review exponential functions PowerPoint.

General Education Math Lesson Plan – Determining Functions Using Regression

Source: http://illuminations.nctm.org/LessonDetail.aspx?ID=L725

Standard*:* 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables.

Using a Calculator for Finding the Equation of a Function

To determine the function of best fit for a set of data, students should recognize which category of function bests fit the data and know how to use technology to obtain a function. This lesson teaches these skills and prepares students for the subsequent lesson(s), in which they will collect their own data.

Instructions on using TI83 and TI84 calculators for regression is available here.

Learning Objectives:

By the end of this lesson, students will:

- Analyze data to determine the type of function that most closely fits the data
- Use a calculator to find the curve of best fit for a set of data

Materials:

- Graphing calculator
- Overhead - Determine the Function
- Activity Sheet - Corn Height
- Activity Sheet - Which Function Is Better?

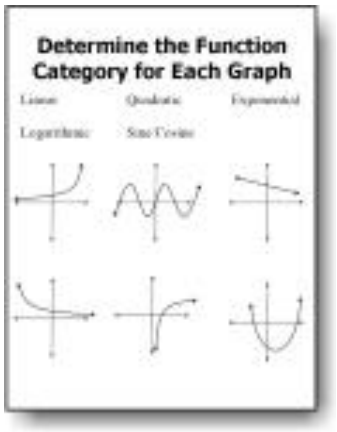
Instructional Plan:

Part 1: Review of Graphs

Students should recognize which type of function a graph represents, so they can use technology to determine the best function for the data set. It is a worthwhile to review these functions since students may not have seen some of them in a while. Also, displaying all the graphs together, as this activity does, can aide students later, as they have to determine the best graph for the data.

Distribute copies of the Determine the Function review sheet, or use it as an overhead. Ask students to match each graph with its proper general description.

Determine the Function review sheet



Part 2: Using the Technology

Whether students are using a calculator or a spreadsheet to do regression, this section is designed to introduce or review the process of finding regression functions.

Present students with the following table of data for the height of a crop of corn over time. This table also appears on Activity Sheet 1.

DAY	HEIGHT (IN CM)
1	1.31
2	2.10
3	4.09
5	6.37
12	14.42

General Steps for Using Regression:

(Specific TI-83 and TI-84 instructions are available here)

1. Enter the data points into the calculator.
2. View the scatter plot of the data.
    - Most likely, the window for viewing the graph is x={-10,10}and y={-10,10}. When the students first enter the data, and look at the scatter plot they will

notice that only 4 of the 5 data points are visible. This is great time to discuss determining the appropriate domain and range for a specific set of data.

3. Determine the best function category for the available data.

    - Students may have different opinions about the best category from the points given. Discuss that in some situations, you may want to try more than one function to determine which is most appropriate for the data.

4. Find the function of best fit for the data.

5. Graph the function of best fit over the data to verify its reasonableness.

Allow some time for the students to answer the questions on Activity Sheet 1. Students may use the graph or the equation to help find their answers.

You may want to ask students some questions about the activity, such as:

- How do you think the calculator finds the equation of the line? Explain how you would get the equation of the line by hand?
- What makes the calculator easy to use?
- What do you have to remember in order to repeat the process?
- What did you write as a response to Question 11?

Consider collecting the Activity Sheet to use for assessment.

Part 3: Finding the Best Function

Sometimes it may not be clear from the given data points which regression to use. This example will elicit a discussion of "best" function.

1. Distribute Activity Sheet 2, Finding the Best Function.

2. Students can work in pairs or small groups to complete the activity sheet.

    - Different groups will get different answers. Most students will determine that the function is exponential because they only see the increase for the positive *x*-values.
    - Encourage students to experiment with other functions until they choose a quadratic form. They should notice that since the negative *x*-values are unknown, it may be tricky to determine the *best* function.

3. Discuss the results a whole class.

    - What function category did you determine was best? Why?

- What would help determine which function is better [more points, or knowing the context or trend for the data]

Here are the answers to the Best Function Activity Sheet.

Once students can determine the function category and use technology to find the specific function, they are ready to explore the next lesson and determine the function resulting from various experiments.

Activity: Create a universally designed version of the above lesson

UDL Planning	**My Ideas**
Representation - adaptations in materials (e.g., adapt for sensory impairments)	Provide students with models of graphs so they can compare provided graphs to the model the "Review of Graphs"; color code calculator to assist in entering data; simplify data sets so student doesn't have so much information to enter.
Expression - how will student show learning (e.g., use of assistive technology; alternative project)	Create response cards or ask questions so that students can respond yes/no pertaining to which graph fits best; use computer software that will allow students to scan and select correct answers using assistive technology.
Engagement - how will student participate in the activity	Students can complete the activities in pairs; create a color coded task analysis for students to follow for entering information in the graphing calculator or how to use the functions to find the equation for the function.

General Education Math Lesson Plan – Determining Functions Using Regression: Lesson Two

Source: http://illuminations.nctm.org/Lesson.aspx?id=4144

Standard: 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables.

What's the Function?
This activity allows students to look for functions within a given set of data. After analyzing the data, students should be able to determine what type of function best represents the data.

Learning Objectives

By the end of this lesson, students will be able to:

- Analyze data to determine the type of function that most closely describes the data
- Demonstrate an understanding of how modifying parameters changes the graphs of functions by writing equations for those functions

Materials

A TI-83 or TI-84 calculator is required for these experiments. The remaining materials are listed by experiment:

1. Activity Sheets
2. Counting M&M's Activity Sheet (Exponential Decay)
    - M&Ms
    - Paper plate
    - Cup or other container
3. Inflating a Balloon Activity Sheet (Power Function)
    - Uninflated balloon (which when inflated is as nearly spherical as possible)
    - Tape measure with centimeters
4. Stacking Cups Activity Sheet (Linear Function)
    - Paper or plastic cups

- Ruler with centimeters

5. Volume of a Box Activity Sheet(Cubic Function)
    - Grid paper cut into 20 cm x 20 cm squares (enough paper for each group to do 3–6 experiments)
6. Weather Patterns Activity Sheet (Sinusoidal, Periodic Function)
    - Internet access to look up local temperature averages OR table of weather data for Washington, DC.

Instructional Plan

Before participating in this lesson, students should understand the relationships between various functions and their graphs. Their knowledge should cover linear, quadratic, higher-degree-polynomial, exponential, and trigonometric functions. They should also be able to use a graphing calculator to edit data, create scatter plots, and draw functions over those plots.

Instructions for using TI-83 and TI-84 calculators for regression are available here.

Structure

Each experiment in this section stands alone. Consider selecting the activities that apply functions that students have already seen.

Options for presenting these activities:

- Use a single experiment and have students perform the activity in pairs or groups. Then, share results as a class.
- Provide each group with the materials for each activity.
- Have different groups perform different activities. Groups can make presentations on their experiment, their graph of the data, and the function determined.
- Set up activity stations. Give students ten minutes to perform the experiment, and maybe start the calculations. After the ten minutes, rotate the groups to the next station until all students have done each activity.

Activity 1: Counting M&M's (Exponential Decay)

Be sure that students have at least 35 M&M's so that they can do the experiment repeatedly. Students will pour the candies onto a plate, and remove the ones that have the “'M”' face up, and putour the rest of the candies back into the cup. Doing this

repeatedly yields an exponential decay function. Remind the student not to eat the candy, and that it has been used by other students in other classes.

Activity 2: Inflating a Balloon with Breath (Power Function)

Students measure the circumference of a balloon while counting the breaths used to fill it up. Encourage students to do some practice breaths before starting to measure the balloon. Get team members involved in determining how big a breath should be, and in helping the student blowing up the balloon to practice being consistent.

(This lesson is adapted from Navigations through Mathematical Connections 9-12.)

Activity 3: Height of Cups (Linear)

Students measure the height of the stack of cups as they put on or take off cups. An alternative procedure is to stack cups on the floor, measuring the distance between the top of the stack and the surface of a desk.

(This lesson is adapted from Navigating through Algebra 6-8.)

Activity 4: Volume of a Box (Cubic Function)

Students find the volume of a box made from a 20 cm x 20 cm grid with squares of equal size cut out from each corner of the grid. Different-sized corners yield boxes of different volumes. Students graph the length against the volume.

- You could extend this activity by asking students to find the length, width, and surface area of each box in addition to its volume. The problem would then include linear, quadratic, and cubic models.

- The post office uses the concept of girth to limit the size of boxes sent in the mail (length + width ≤ 108 inches). Ask students to find the box with the greatest volume for the least surface area that meets postal regulations.

Activity 5: Weather Patterns (Sinusoidal or Periodic)

Students gather data about local temperatures over a year. To minimize the data but still give meaningful results, students can use the temperature on the 1st and 15th of each month.

- This information reduces the domain to 24 values but still yields a fairly smooth curve.

- You can find the average monthly temperature for many cities in almanacs on the Internet. Alternatively, give students this table of weather data for Washington, DC. The temperatures in this table reflect the *average* high temperatures for the dates within one week of the listed date. For example, the temperature listed for

Jan 15 reflects the average high temperature for all days from Jan 8 through Jan 22.

- Alternatively, all of the temperature data for Washington, DC, in 2011 can be found in the Washington Temperatures spreadsheet (Excel).

- This activity works best when the calculator is in radian mode.

Activity: Create a universally designed version of the above lesson

UDL Planning	**My Ideas**
Representation - adaptations in materials (e.g., adapt for sensory impairments)	
Expression - how will student show learning (e.g., use of assistive technology; alternative project)	
Engagement - how will student participate in the activity	