## Mathematics Instructional Families Patterns, Relations and Functions

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## ncsc

National Center and State Collaborative

The National Center and State Collaborative (NCSC) is applying the lessons learned from the past decade of research on alternate assessments based on alternate achievement standards (AA-AAS) to develop a multi-state comprehensive assessment system for students with significant cognitive disabilities. The project draws on a strong research base to develop an AA-AAS that is built from the ground up on powerful validity arguments linked to clear learning outcomes and defensible assessment results, to complement the work of the Race to the Top Common State Assessment Program (RTTA) consortia.

Our long-term goal is to ensure that students with significant cognitive disabilities achieve increasingly higher academic outcomes and leave high school ready for postsecondary options. A well-designed summative assessment alone is insufficient to achieve that goal. Thus, NCSC is developing a full system intended to support educators, which includes formative assessment tools and strategies, professional development on appropriate interim uses of data for progress monitoring, and management systems to ease the burdens of administration and documentation. All partners share a commitment to the research-to-practice focus of the project and the development of a comprehensive model of curriculum, instruction, assessment, and supportive professional development. These supports will improve the alignment of the entire system and strengthen the validity of inferences of the system of assessments.

This work was developed as part of the National Center and State Collaborative and supported by a grant from the Department of Education (PR/Award \#: H373X100002, Project Officer, Susan.Weigert@Ed.gov). The contents do not necessarily represent the policy of the U.S. Department of Education, and no assumption of endorsement by the Federal government should be made.

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These materials and documents were developed under the National Center and State Collaborative (NCSC) General Supervision Enhancement Grant and are consistent with its goals and foundations. Any changes to these materials are to be consistent with their intended purpose and use as defined by NCSC.

This document is available in alternative formats upon request.

## nCSC

National Center and State Collaborative
NCSC is a collaborative of 15 states and five organizations.
The states include (shown in blue on map): Arizona, Connecticut, District of Columbia, Florida, Georgia, Indiana, Louisiana, Nevada, Pacific Assessment Consortium (PAC-6) ${ }^{1}$, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, and Wyoming.

Tier II states are partners in curriculum, instruction, and professional development implementation but are not part of the assessment development work. They are (shown in orange on map): Arkansas, California, Delaware, Idaho, Maine, Maryland, Montana, New Mexico, New York, Oregon, and U.S. Virgin Islands.


[^0]The five partner organizations include: The National Center on Educational Outcomes (NCEO) at the University of Minnesota, The National Center for the Improvement of Educational Assessment (Center for Assessment), The University of North Carolina at Charlotte, The University of Kentucky, and edCount, LLC.


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# Mathematics Instructional Families Patterns, Relations and Functions 

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January 2013

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## View of Learning Targets and Families across Grades

## Distribution of Instructional Families: Patterns, Relations and Functions



# View of Learning Targets, Families, and CCCs by Grade-band 

## Overview of CCCs: Patterns, Relations and Functions

| Representing and Proble | Modeling Descr | Describing and Extending Patterns |  | Problem | olving and Using ariables |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (K-4) Elementary School Learning Targets |  |  |  |  |  |
| E.PRF-1 Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change. |  |  |  |  |  |
| E.PRF-2 Give examples, interpret, and analyze repeating and growing patterns and functions involving the four basic operations |  |  |  |  |  |
| Grade K | Grade 1 | Grade 2 |  | Grade 3 | Grade 4 |
| K.PRF.1b1 Using objects or pictures respond appropriately to "add _" $\qquad$ and "take away $\qquad$ K.OA. 1 | 1.PRF.1b3 Using objects or pictures respond appropriately to "add $\qquad$ " and "take away $\qquad$ 1.OA. 1 | 2.PRF.1c3 Solve one or two step addition and subtraction problems, and add and subtract within 100, using objects, drawings, pictures 2.OA. 1 | 3.PRF. 10 model m division stideur up to 5 g objects in interpret 3.OA. 1 3.OA. 2 | d1 Use objects to ultiplication and situations involving roups with up to 5 in each group and the results | 4.PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and interpret the results 3.OA. 1 |
| K.PRF.1c1 Solve one step addition and subtraction word problems, and add and subtract within 10 using objects, drawings, pictures K.OA. 2 | 1.PRF.1c2 Solve one step addition and subtraction word problems where the change or result is unknown $(4+=7)$ or $(4+3=\ldots)$, within 20 using objects, drawings, pictures 1.OA. 1 | 2.PRF.1c4 Use pictures, drawings or objects to represent the steps of a problem 2.OA. 1 | 3.PRF. 2 d of operatio to multipl 3.OA. 5 | d2 Apply properties tions as strategies ly and divide | 4.PRF. 1 f3 Apply the distributive property to solve problems with models 3.MD.7c |
| K.PRF.2a1 Describe or select the repeating pattern using objects or pictures (AB or ABC) No CCSS linked | 1.PRF.2a4 Use a number line to extend the numerical patterns that grow at a constant rate $(2,4,6,8)$ No CCSS linked | 2.PRF.2a6 Use a number line to extend the numerical patterns that grow at a constant rate $(2,4,6,8)$ No CCSS linked | 3.PRF. 1 e for a num increase 3.OA. 9 | 1 Describe the rule erical pattern (e.g., by 2,5 or 10) | 4.PRF.2d3 Generate a pattern when given a rule and word problem 4.OA. 5 |
| K.PRF.2a2 Extend a repeating pattern using objects or pictures (AB or ABC) <br> No CCSS linked | 1.PRF.2b2 Create a growing pattern using numbers or objects No CCSS linked | 2.PRF.2b3 Use a number line to extend arithmetic patterns that are decreasing | 3.PRF. 1 <br> the 3 nex numerical numbers 10 $\text { 3.OA. } 9$ | 2 Select or name xt terms in a pattern where increase by 2, 5 or | 4.PRF.2e1 Extend a numerical pattern when the rule is provided 4.OA. 5 |
| K.PRF.2a3 Extend a repeating numerical $A B$ pattern No CCSS linked | 1.PRF.2c1 Identify the rule of a given arithmetic pattern No CCSS linked | 2.PRF.2c2 Identify the rule of arithmetic patterns that are increasing No CCSS linked | 3.PRF.2d <br> multiplica real-world 3.OA. 9 | d1 Identify ation patterns in a d setting | 4.PRF.1e3 Solve multiplicative comparisons with an unknown using up to 2-digit numbers with |


|  |  |  | information presented in a <br> graph or <br> word problem |  |
| :--- | :--- | :--- | :--- | :--- |
| K.PRF.2b1 Create a <br> repeating pattern using <br> objects, pictures, or <br> numbers <br> No CCSS linked |  | 2.PRF.2c3 Identify the rule <br> of arithmetic patterns that <br> are decreasing <br> No CCSS linked |  |  |
| K.PRF.1b2 Communicate <br> answer after adding or <br> taking away |  | 2.PRF.1c5 Write or select <br> an equation representing <br> the problem and its solution <br> K.OA.1 |  |  |

## Overview of CCCs: Patterns, Relations and Functions

| Describing and Extending | Patterns | Problem Solving and Using Variables |  | Proportional Relationships and Graphing |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (5-8) Middle School Learning Targets |  |  |  |  |  |
| M.PRF-1 Describe and compare situations that involve change and use the information to draw conclusions: <br> - Model contextual situations using multiple representations; <br> - Calculate rates of change for real-world situations (constant) |  |  |  |  |  |
| M.PRF-2 Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions |  |  |  |  |  |
| Grade 5 |  | ade 6 | Grad |  | Grade 8 |
| 5.PRF.1b1 Given 2 patterns involving the same context (e.g., collecting marbles) determine the $1^{\text {st }}$ 5 terms and compare the values 5.OA. 3 | 6.PRF.1d1 step linear 6.EE. 7 | real-world single ions | 7.PRF. 1 g 1 Solve step problems us 7.EE. 3 | multi numbers | 8.PRF. 1 g3 Solve linear equations with 1 variable 8.EE. 7 |
| 5.PRF.2a1 Generate a pattern that follows the provided rule $\text { 4.OA. } 5$ | 6.PRF.2a2 represent expression problems 6.EE. 6 | ariable to s and write solving real-world | 7.PRF. 1 g 2 Use va represent quantitie or mathematical pr construct simple e inequalities to solv reasoning about th 7.EE. 4 | I-world <br> nd <br> and <br> sy <br> es | 8.PRF.1e2 Represent proportional relationships on a line graph 8.EE. 5 |
| 5.PRF.1b2 When given a line graph representing two arithmetic patterns, identify the relationship between the two $\text { 5.OA. } 3$ | 6.PRF.2a3 represent two world proble relationship 6.EE. 9 | variables to untities in a realat change in ne another | 7.PRF.2d1 Solve leading to inequali $+q>r$ or $p x+q<$ $r$ are specific ration 7.EE 4b | ems form px $p, q$, and s | 8.PRF.1f2 Describe or select the relationship between the two quantities given a line graph of a situation 8.EE. 5 |
| 5.PRF.2b1 Generate or select a comparison between two graphs from a similar situation 5.OA-3 | 6.PRF.1a2 not the quo decrease b 5.NF. 5 | mine whether or will increase or on the divisor | 7.PRF. 1 e2 Repres relationships on a 7.RP.2b | ortional | 8.PRF.2c1 Given two graphs, describe the function as linear and not linear $\text { 8.F. } 3$ $\text { 8.F. } 5$ |
| 5.PRF.1a1 Determine whether the product will increase or decrease based on the multiplier 5.NF. 5 | 6.PRF.1c1 relationship for a given 6.RP. 1 | ribe the ratio een two quantities ion | 7.PRF. 1 f1 Use pro relationships to so percent problems situations. 7.RP. 3 |  | 8.PRF.2e1 Distinguish between functions and non-functions, using equations, graphs or tables No CCSS linked |



## Overview of CCCs: Patterns, Relations and Functions

## Problem Solving and Using Variables <br> Proportional Relationships and Graphing <br> (9-12) High School Learning Targets

H.PRF-1 Approximate, calculate, model, and interpret change:

- Use graphical and numerical data resulting from complex situations;
- Model complex real-world phenomena to make predictions and provide explanations
H.PRF-2 Use trends and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.


## HS

H.PRF.2a1 Translate an algebraic expression into a word problem A.SSE. 1
H.PRF.2b1 Translate a real-world problem into a one variable equation
A.CED. 1
H.PRF.2b2 Solve equations with one or two variables using equations or graphs
A.REI. 1
A.REI. 3
A.CED. 2
H.PRF. 1 a1 Interpret the rate of change using graphical representations
S.ID. 7
H.PRF.1b1 In a linear situation using graphs or numbers, predicts the change in rate based on a given change in one variable
F.LE. 1 b
H.PRF.1c1 Select the appropriate graphical representation of a linear model based on real-world events
F.LE. 1
H.PRF. 2c1 Make predictions based on a given model
F.LE. 3

## View by Instructional Families, LPF, and CCSS Domains

## Instructional Families within LPF Strand: Patterns, Relations and Functions

| CCSS Domain: Operations and Algebraic Thinking; Measurement and Data | CCSS Domain: Operations and Algebraic Thinking | CCSS Domain: Operations and Algebraic Thinking; Expressions and Equations; Seeing Structure in Expressions; Creating Equations; Reasoning with Equations and Inequalities | CCSS Domain: Operations and Algebraic Thinking; Number Operations-Fractions; Ratios and Proportional Relationships; Expressions and Equations; Functions; Interpreting Categorical and Quantitative Data; Linear, Quadratic and Exponential Models |
| :---: | :---: | :---: | :---: |
| Representing and Modeling Problems | Describing and Extending Patterns | Problem Solving and Using Variables | Proportional Relationships and Graphing |
| K.PRF.1b1 Using objects or pictures respond appropriately to "add __" and "take away K.OA. 1 | K.PRF.2a1 Describe or select the repeating pattern using objects or pictures (AB or ABC) <br> No CCSS linked | K.PRF.1b2 Communicate answer after adding or taking away K.OA. 1 | 5.PRF. 1b2 When given a line graph representing two arithmetic patterns, identify the relationship between the two $\text { 5.OA. } 3$ |
| K.PRF.1c1 Solve one step addition and subtraction word problems, and add and subtract within 10 using objects, drawings, pictures K.OA. 2 | K.PRF.2a2 Extend a repeating pattern using objects or pictures (AB or $A B C$ ) <br> No CCSS linked | 2.PRF.1c5 Write or select an equation representing the problem and its solution 2.OA. 1 | 5.PRF.2b1 Generate or select a comparison between two graphs from a similar situation 5.OA-3 |
| 1.PRF.1b3 Using objects or pictures to respond appropriately to "add _" and "take away _-" 1.OA. 1 | K.PRF.2a3 Extend a repeating numerical $A B$ pattern No CCSS linked | 4.PRF.1e3 Solve multiplicative comparisons with an unknown using up to 2-digit numbers with information presented in a graph or word problem (e.g., an orange hat cost $\$ 3$. A purple hat cost 2 times as much. How much does the purple hat cost? [ $3 \times 2=$ p]) 4.OA. 2 | 5.PRF.1a1 Determine whether the product will increase or decrease based on the multiplier 5.NF. 5 |
| 1.PRF.1c2 Solve one step addition and subtraction word problems where the change or result is unknown ( $4+=7$ ) or ( $4+3=\ldots$ ), within 20 using objects, drawings, pictures 1.OA. 1 | K.PRF.2b1 Create a repeating pattern using objects, pictures, or numbers <br> No CCSS linked | 6.PRF.1d1 Solve real world single step linear equations 6.EE. 7 | 6.PRF.1a2 Determine whether or not the quotient will increase or decrease based on the divisor 5.NF. 5 |


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| :---: | :---: | :---: | :---: |
| Representing and Modeling Problems | Describing and Extending Patterns | Problem Solving and Using Variables | Proportional Relationships and Graphing |
| 2.PRF. 1 c 3 Solve one or two step addition and subtraction problems, and add and subtract within 100, using objects, drawings, pictures 2.OA. 1 | 1.PRF.2a4 Use a number line to extend the numerical patterns that grow at a constant rate $(2,4,6,8)$ No CCSS linked | 6.PRF.2a2 Use variable to represent numbers and write expressions when solving real world problems 6.EE. 6 | 6.PRF.1c1 Describe the ratio relationship between two quantities for a given situation 6.RP. 1 |
| 2.PRF.1c4 Use pictures, drawings or objects to represent the steps of a problem 2.OA. 1 | 1.PRF.2b2 Create a growing pattern using numbers or objects No CCSS linked | 6.PRF.2a3 Use variables to represent two quantities in a realworld problem that change in relationship to one another 6.EE. 9 | 6.PRF.1c2 Represent proportional relationships on a line graph 6.RP. 2 |
| 3.PRF.1d1 Use objects to model multiplication and division situations involving up to 5 groups with up to 5 objects in each group and interpret the results <br> 3.OA. 1 <br> 3.OA. 2 | 1.PRF.2c1 Identify the rule of a given arithmetic pattern No CCSS linked | 7.PRF. 1 g 1 Solve real world multi step problems using whole numbers 7.EE. 3 | 6.PRF.2a4 Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation 6.EE. 9 |
| 3.PRF.2d2 Apply properties of operations as strategies to multiply and divide $\text { 3.OA. } 5$ | 2.PRF.2a6 Use a number line to extend the numerical patterns that grow at a constant rate ( $2,4,6,8$ ) No CCSS linked | 7.PRF. 1 g 2 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities 7.EE. 4 | 6.PRF.2b2 Using provided table with numerical patterns, form ordered pairs $\text { 5.OA. } 3$ |
| 4.PRF.1d2 Use objects to model multiplication and division situations involving up to 10 groups with up to 5 objects in each group and | 2.PRF.2b3 Use a number line to extend arithmetic patterns that are decreasing No CCSS linked | 7.PRF.2d1 Solve word problems leading to inequalities of the form px $+q>r$ or $p x+q<r$, where $p, q$, and $r$ are specific rational numbers | 6.PRF.2b3 Complete a statement that describes the ratio relationship between two quantities 6.RP. 1 |


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| :---: | :---: | :---: | :---: |
| Representing and Modeling Problems | Describing and Extending Patterns | Problem Solving and Using Variables | Proportional Relationships and Graphing |
| interpret the results 3.OA. 1 |  | 7.EE 4b |  |
| 4.PRF. $1 \mathrm{f3} 3$ Apply the distributive property to solve problems with models <br> 3.MD.7c | 2.PRF.2c2 Identify the rule of arithmetic patterns that are increasing No CCSS linked | 8.PRF. 1 g3 Solve linear equations with 1 variable 8.EE. 7 | 6.PRF.2b4 Determine the unit rate in a variety of contextual situations 6.RP. 2 |
|  | 2.PRF.2c3 Identify the rule of arithmetic patterns that are decreasing No CCSS linked | H.PRF.2a1 Translate an algebraic expression into a word problem A.SSE. 1 | 6.PRF.2b5 Use ratios and reasoning to solve real-world mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations) 6.RP. 3 |
|  | 3.PRF.1e1 Describe the rule for a numerical pattern (e.g., increase by 2,5 or 10) <br> 3.OA. 9 | H.PRF.2b1 Translate a real-world problem into a one variable equation A.CED. 1 | 7.PRF.1e2 Represent proportional relationships on a line graph 7.RP.2b |
|  | 3.PRF. 1 e2 Select or name the 3 next terms in a numerical pattern where numbers increase by 2,5 or 10 3.OA. 9 | H.PRF.2b2 Solve equations with one or two variables using equations or graphs <br> A.REI. 1 <br> A.REI. 3 <br> A.CED. 2 | 7.PRF.1f1 Use proportional relationships to solve multi step percent problems in real world situations. <br> 7.RP. 3 |



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| :---: | :---: | :---: | :---: |
| Representing and Modeling Problems | Describing and Extending Patterns | Problem Solving and Using Variables | Proportional Relationships and Graphing |
|  |  |  | 8.PRF.2e2 Identify the rate of change (slope) and initial value ( $y$ intercept) from graphs 8.F. 4 |
|  |  |  | 8.PRF.2e3 Given a verbal description of a situation, create or identify a graph to model the situation 8.F. 5 |
|  |  |  | 8.PRF.2e4 Given a graph of a situation, generate a description of the situation 8.F. 5 |
|  |  |  | 8.DPS.2g2 Interpret the slope and the $y$-intercept of a line in the context of a problem 8.SP. 3 |
|  |  |  | H.PRF.1a1 Interpret the rate of change using graphical representations S.ID. 7 |


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| :---: | :---: | :---: | :---: |
| Representing and Modeling Problems | Describing and Extending Patterns | Problem Solving and Using Variables | Proportional Relationships and Graphing |
|  |  |  | H.PRF. 1 b1 In a linear situation using graphs or numbers, predicts the change in rate based on a given change in one variable (e.g., I have been adding sugar at a rate of 1 T per cup of water. What happens to my rate if I switch to 2T of sugar for every cup of water?) <br> F.LE. 1 b <br> H.PRF. 1 c1 Select the appropriate graphical representation of a linear model based on real world events F.LE. 1 <br> H.PRF. 2c1 Make predictions based on a given model (for example, a weather model, data for athletes over years) F.LE. 3 |


[^0]:    ${ }^{1}$ The Pacific Assessment Consortium (including the entities of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia, Guam, Republic of Palau, and Republic of the Marshall Islands) partner with NCSC as one state, led by the University of Guam Center for Excellence in Developmental Disabilities Education, Research, and Service (CEDDERS).

