Mathematics:
Measurement
9-10

NCSC Sample Instructional Unit

Investigating Measurement in the Real World

National Center and State Collaborative

The contents of this product were developed under a grant from the Department of Education (PR/Award #:H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

This instructional unit was originally developed through the National Alternate Assessment Center (NAAC), a five year project funded under the United States Department of Education, Office of Special Education Programs (OSEP), and is covered under the original copyright of 2010. The authors disclosed receipt of the following financial support for the research and/or authorship of this material: U.S. Department of Education Office of Special Education Programs Grant No. H3244040001. Opinions expressed in the document do not necessarily reflect those of the funders.

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Unit Key Vocabulary

Area. The space that covers a two dimensional figure which is measured in square units.

Centimeter. A metric unit of length equal to one hundredth of a meter.

Conversion. A change from one unit or unit system to another. For example, inches to feet, inches to centimeters.

Dimensions. The length, height, and width of a figure.

Foot. A standard unit of length in the US Customary system of measurement equal to 12 inches.

Inch. A standard unit of length in the US Customary system of measurement equal to 1/12 of a foot.

Length. A measurement of the distance from one point to another.

Meter. The basic Standard International unit of length.

Perimeter. The distance around a closed figure which is measured in units.

Proportion. Equality between two ratios. For example, 8:12 and 4:6.

Ratio. A compared relationship between two numbers. For example, 2:1.

Rectangle. A four-sided figure with four right angles.

Similar Rectangles. Rectangles of different sizes with all corresponding angles congruent and all corresponding sides are proportional.

Unit of Measure. A defined and adopted magnitude of a physical quantity such weight, length, temperature.

Unit Rate. A ratio with the second term being a unit of one. For example, miles per hour; dollars per hour, etc.

Width. Measurement of the distance from one side or edge to the opposite side or edge.

Yards. Measurement of length in which one yard is equal to 3 feet or 36 inches.

Unit Standards Overview

Common Core State Standard:

HSN-Q Number and Quantity
- **HSN-Q.A** Reason quantitatively and use units to solve problems.
  - **HSN-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
  - **HSN-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.

HSG-MG Geometry
- **HSG-MG.A** - Apply geometric concepts in modeling situations.
  - **HSG-MG.A.1** Use geometric shapes, their measures, and their properties to describe objects.
  - **HSG-MG.A.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
  - **HSG-MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Learning Progressions Frameworks Progress Indicator

H.ME.1a Making decisions about units and scales that are appropriate for problem-solving situations involving mathematics within or across mathematics disciplines or real-world context.
H.ME.1b Investigating the results when linear dimensions of objects change by some factor (e.g., area and volume change disproportionately: area in proportion to the square of the factor and volume in proportion to its cube)

Instructional Family: Problem solving using measurement process.

Core Content Connectors addressed:
- **H.ME.1a1** Determine the necessary unit(s) to use to solve real world problems.
- **H.ME.1a2** Solve real world problems involving units of measurement.
- 3.ME.1g1 Identify a figure as getting larger or smaller when the dimensions of the figure changes.

Instructional Family: Perimeter, Area, Volume

Core Content Connectors addressed:
- **H.ME.1b1** Describe the relationship between the attributes of a figure and the changes in the area or volume when 1 attribute is changed.
- 8.ME.1e2 Compare area and volume of similar figures.
- 6.ME.1a1 Identify the appropriate formula (i.e., perimeter, area, volume) to use when measuring for different purposes in a real life context.
- 4.ME.2h1 Apply the formulas for area and perimeter to solve real world problems.
- 3.ME.2h Use addition to find the perimeter of a rectangle.

Instructional Family: Scaling and Unit conversion

Core Content Connectors addressed:
- **H.ME.2b5** Apply the formula of geometric figures to solve design problems (e.g., designing an object or structure to satisfy physical restraints or minimize cost)
- **H.ME.2b1** Determine the dimensions of a figure after dilation
- 8.ME.1e1 Describe the changes in surface area, area, and volume when the figure is changed in some way (e.g., scale drawings)
- 7.ME.1d1 Solve problems that use proportional reasoning with ratios of length and area
- 7.PRF.1e1 Determine unit rates associated with ratios of lengths, areas, and other quantities measured in like units
- 5.ME.2a1 Solve problems involving conversions of standard measurement units when finding area, volume, time lapse, or mass.
- 5.ME.1b2 Convert standard measurements of length.

Grade Span: 9-10	**Content Area:** Mathematics – Measurement Investigating Measurement in the Real World
Lesson 1 of the Unit	**Approximate Time Needed:** 50 minutes

Objective: Student will make decisions about units and scales that are appropriate for problem solving situations involving mathematics within mathematics or across disciplines or contexts.
Essential Question: What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?

Lesson 1: Materials

Materials Needed:

Using grid paper or a Geoboard, provide students with rectangles having different dimensions, some of which are similar. Some of the rectangles should have the same perimeter but different areas.

For example:

- 9 x 12, P = 42; A = 108
- 8 x 13 P = 42; A = 104
- 6 x 15 P = 42; A = 90

Some should have the same area but different perimeters.

For example:

- 4 x 5; A = 20; P = 18
- 10 x 2; A = 20; P = 24

Make chart as used in practice note below to reinforce skills.

- Modify chart to include picture and/or representations of headings.
- Provide chart in electronic format for access with switch or alternate keyboard.

Materials needed:

- Grid paper
  - Grid paper with raised lines
  - Grid paper created on overhead transparencies for use with light boards
  - Virtual grid paper
- Geoboards
  - Virtual Geoboards

- Calculator
- Paper and pencils
- Ruler
- Yardstick
- Foldable ruler
- Conversion charts(for inches to feet, feet to yards, centimeters to meters)

See Resources: See Lesson 1 Resources for example exercises/images.

Lesson Vocabulary
Area
Centimeter
Foot
Inch
Length
Meter
Perimeter
Rectangles
Similar Rectangles
Width
Yards

Lesson 1: Introduction – 10 minutes

A. Activate Previous Knowledge

1. Lead a short discussion about how to find perimeter and area of rectangles.
   - Review the concepts of perimeter and area.
   - Discuss how these concepts are used in real life examples.
     - Example 1: A runner is practicing by running along the fence line of a parking lot.
       – Is he running the perimeter of the parking lot or is he running the area?
     - Example 2: The school is getting new carpet in the classroom.
       – Will the workers need to figure out the area of the classroom or the perimeter?
2. Break class into small groups to answer exercises.
3. Using figures (rectangles and squares) drawn on grid paper or formed on Geoboards, find the perimeters and areas.
4. Remind students that answers should/must include the appropriate units of measure.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc. to complete exercises.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide picture and/or tactile and/or object representations of relevant vocabulary paired with the written word as it is mentioned during presentation or discussion for rectangle, area, perimeter as well as the meanings of each word.
2. Create math journals to record vocabulary, formulas, and notes.
3. Provide the formulas for area and perimeter as the concepts of each are discussed.
4. During discussion, provide picture representation of real world uses for area and perimeter.
5. As students work in small groups or pairs, ensure they have a means for gaining their group members' or partner's attention and a means for contributing to the discussion.

6. Students may use their math journals or a graphic organizer to collect/store information gathered during group.

7. To find area and perimeter, use grid paper, count/mark/tally each unit along the length of the figure to determine length and count/mark/tally each unit along the width of the figure to determine the width.

8. Use the formulas to determine area and perimeter.
    - A list of formulas may be used by the student as a reference.

9. Student may be presented with manipulatives of a unit and the rectangle drawn on grid paper.
    - Students determine area and perimeter by placing the manipulative units on each unit around the rectangle on the grid paper to demonstrate perimeter as well as within the rectangle to demonstrate area.

    - Using manipulatives may be demonstrated electronically by using a computer program or PowerPoint to count units virtually to determine area and perimeter.
        - Each time the student hits the switch, the computer program counts each unit around the rectangle to determine perimeter.
        - To determine area, each time the student hits the switch, the program counts the units within the rectangle or for larger numbers, highlights a row or column of units and skip counts by 5s to determine the total number of units.

See Resources: See PowerPoint, Slides 1 and 2.

10. As answers are reviewed, be sure to reference the appropriate units of measure. For example, if students determine the perimeter of a 3inch by 4inch figure is 14, reply, "That is correct. It is 14 inches." If they determine the area is 12, reply, "That is correct. It is 12 inches square."
    - Remind students to record the appropriate unit.
    - Model how to write the appropriate units.
    - Present students with an alternative representation of unit to record in their math journals or graphic organizers.

Important Note for Communicators Considered Pre-Symbolic: Be sure students have a way to attain peer attention as well as to share and receive information. Limit measurements to one type: standard or metric unit.

B. Establish Goals/Objectives for the Lesson

Inform students that in this lesson they will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Convert units using standard/known conversion units.
2. Use appropriate known formulas for the area.
3. Solve multistep problems involving one unit of measure.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement*:* Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

Inform students of expected outcomes.

- Provide keys words paired with symbols/images/tactile representations.
- Provide the key words in the lesson objectives paired with images or tactile representations, record into mathematics journals, or students may use an electronic picture writer paired with text-to-speech to record the lesson's objectives.
- Provide photographs, models or tactile representations of examples of situations in which these concepts are used.

Lesson 1: Body – 15 minutes

Direct Instruction and/or Facilitation of the Lesson

1. Ask students:
    a. "For two rectangles of the same (similar) shape, how do their sizes compare?"
    b. "How do their perimeters compare?"
    c. "How do their areas compare?"

See Resources: Lesson 1, Pages 15 and 16.

2. Identify rectangles that have the same areas.
    a. Ask students: "Do they have the same perimeter?"

See Resources: Lesson 1, Page 17.

3. Lead a discussion on how to find perimeter and area of rectangles when units are different. (e.g., a rectangle measures 8 inches wide, 2 feet long or 80 cm wide, 1 meter long).

4. Review converting units:
    a. From inches to feet
    b. From feet to inches
    c. From feet to yards
    d. From yards to feet
    e. From centimeters to meters
    f. From meters to centimeters

See Resources: Lesson 1, Page 18.

Multiple means of representation: Use models and/or drawings during large group instruction; allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter or provide options for using manipulatives and/or computer models.

Multiple means of engagement*:* Allow students to use paper/pencil, manipulatives, computer, etc. to complete exercises. Present information within the context of students' interests such as pets, gardening, new bedroom floor plan, etc.

Additional Considerations for Emerging Readers

1. Provide picture representations of relevant vocabulary: similar rectangle, area, perimeter as well as the meanings of each word.

2. Provide students with grid paper with two similar rectangular figures printed on it or grid paper with manipulatives of the figures.

- Ensure students have a means for sharing how the rectangles are the same and/or different. Students use their math journals or a graphic organizer to record information about the attributes and measurements of each rectangle. Students use the same strategies as were used to determine area and perimeter in the introduction.
- Students share how the perimeters are the same or different using the information recorded in the journals or the graphic organizer.
- Students share how the areas are the same or different using the information recorded in the journals or the graphic organizer.

3. Demonstrate how ratios are used to compare the area of each figure and the perimeter of each figure.

4. Demonstrate how ratios are used to compare the area of each figure and the perimeter of each figure.

See Example: Lesson 1, Body 1, Similar Figures.

5. Provide students with two rectangles that have different measurements but the same area drawn on grid paper or as manipulatives of the figures.
   - Students verify that the area is the same and determine the perimeter of each figure.

See Example: Lesson 1, Body 2, Same Area.

6. Provide students with picture representations of relevant vocabulary for discussion: inches, feet, yard, centimeter, and meter as well as the meanings of each word.

7. When discussing different measurements within the same system (inches, feet, and yards, or centimeters and meters), present students with picture/object/tactile representations of the different measures.

8. To build understanding of the relationship between the different measurements, have students measure:
   - a book using inches marked on a ruler;
   - a table using feet while attending to how many inches that would be;
   - the length of the chalkboard/ whiteboard using a yardstick while attending to how many feet that would be.

9. Repeat measurements using centimeters and meters.

10. Emphasize the difference in size of object in relation to the size of unit used.

11. Students measure a given piece of paper that is 8 inches wide and 2 feet long.
    - Allow students to explore other units that can be used.
    - Discuss how the paper can be measured using the same unit of measure (i.e., Inches, 8 inches wide and 24 inches long).

12. Provide students with a calculator, the formulas, and task-analyzed steps for converting from one unit to another. Or provide students with a conversion chart with which to match the measured unit to a converted unit.
    - As a whole class or in small groups, work together to convert the units of measure for a rectangle that measures 8 inches wide and 2 feet long.
    - Students use the conversion formulas or create the figures on grid paper.
    - Students count the units on the grid paper to determine the conversion.

See Example: Lesson 1, Body 4, Converting Units.

Additional Considerations for Emerging Communicators

1. Provide picture and tactile representations of relevant vocabulary: similar rectangle, area, perimeter as well as the meanings of each word.
2. Using manipulatives or a computer program or PowerPoint that contains 2 similar figures (Figure A and Figure B) in a grid, students determine the area of Figures A and B by counting or using one-to-one correspondence to determine the number of units within the figure (area) and number of units around the figure (perimeter).
    - Students compare the areas by moving the smaller figure (Figure B) into the larger figure (Figure A) to see how many are needed to completely cover the larger figure.
    - This activity demonstrates how many times bigger Figure A is than Figure B.

See Example: PowerPoint Lesson 1, Slides 3 and 4.

3. Using a computer program or PowerPoint that contains 2 figures (Figure A and Figure B) with the same area in a grid, students use a switch to determine the area for each figure and verify that they are the same.
    a. Students determine if the perimeters are also the same.

See Example: PowerPoint Lesson 1, Slides 5 and 6.

4. Provide picture and tactile/object representations of relevant vocabulary: inches, feet, yard, centimeter, meter as well as the meanings of each word.

5. When discussing different measurements within the same system (inches, feet, and yards, or centimeters and meters), present students with tactile representations or objects of the different measures.
    - Students work with peer partners to measure a book using inches marked on a foldable ruler so students feel the inches.
    - Students measure a table using feet while attending to how many inches that would be. Again use the foldable ruler.
    - To build understanding of the relationship between the different measurements, students measure the length of the chalkboard/whiteboard using a yardstick while attending to how many feet that would be.

- Emphasize the difference in the size of an object in relation to the size of the unit used.
- Students measure a given piece of paper that is 8 inches wide and 2 feet long.
- To emphasize the inches within the foot, use a foldable ruler to show students that the piece of paper can also be measured as 8 inches wide and 24 inches long.

6. As a whole class or in small groups, work together to convert the units of measure for a rectangle that measures 8 inches wide and 2 feet long.

a. Provide students with the figure represented on grid paper.

- Tactilely represent the figure with the original units of inches across and feet down as well as with a rectangle with all measurements converted to inches.
- Represent the different units of measure (1foot and 12 inches) tactilely with different thicknesses, so when students are tactilely scanning the inches, they feel the difference when a foot has been reached.
- Students use a computer program with an alternate keyboard or multi-switch access with each tactile representation on the keyboard or one per switch.
- Students locate the original figure and press it so it is represented on the computer screen.
- The computer program states the original measurements (8 inches by 2 feet), and students locate and press the tactile representation that has been converted to inches.
- The figure is displayed on the screen, and the computer states the converted measurements (8 inches by 24 inches).

See Example: PowerPoint Lesson 1, Slide 7.

Important Note for Communicators Considered Pre-Symbolic: Work with only one system: standard or metric units.

Lesson 1: Practice – 20 minutes

1. In small groups, students work on a variety of problems using different given dimensions such as:
    - Alex has 140 feet of fencing to place around a rectangular garden he is making. He wants the area of the garden to be as large as possible. What should the length and width of the garden be?
    - Give each student the chart below.
    - Students may use models to explore the various possibilities and complete the chart.

Rectangle	Length	Width	Perimeter	Area
A	40 ft	30 ft	140 ft	1200 ft^2

See Resources: Pages 20 and 21.

2. Bring the whole group back together.
3. Ask one student from each group to discuss the results.

4. After reviewing students' results, help students realize that when a perimeter is 140 units, length plus width always equals 70 units, but length times width varies.
5. Make a chart with students of the possible lengths and widths of a rectangle in increments of 5 when the perimeter is 140 units. Use information from their charts and organize it in order from smallest length to largest length.

	L + W	L x W
70 units =	5 units + 65 units	5 units x 65 units = 325 $units^2$
70 units =	10 units + 60 units	10 units x 60 units = 600 $units^2$
70 units =	20 units + 50 units	20 units x 50 units = 1000 $units^2$
70 units =	35 units + 35 units	35 units x 35 units = 1225 $units^2$
70 units =	50 units + 20 units	50 units x 20 units = 1000 $units^2$

6. Demonstrate to students that as the dimensions change, the area gets larger, reaching the highest value, and then gets smaller. Students should conclude that when asked to find the largest area when given a perimeter, a square would always have the greatest area.

Multiple means of representation: Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models.

Multiple means of engagement*:* Ensure each student is actively involved in the small groups. Present different problems related to students' interests. Use questioning to encourage students to explain their strategies in their groups.

Additional Considerations for Emerging Readers

1. Provide picture representations of the word problem to students as it is being read.
    - Present word problem electronically using text-to-speech.
    - Label each column of the table using picture symbols and include measurements for length and width needed for a perimeter of 140 ft.
    - Remind students how to determine perimeter and area.
        - Provide formulas and task-analyzed steps for using the formulas:
            - Perimeter = L+L+W+W or 2L + 2W
            - Area = L x W
            - Perimeter = 140ft: 2L+2W = 140ft or L+W = 70ft
        - Provide students with models of the various fenced-in areas (rectangles).
            - Students use the given formula and calculators to determine area.
        - Students use the grid paper to identify the rectangle that provides the most area.

2. For steps 3-6, students discuss results by presenting the length and width of the rectangle with the largest area that they discovered in their group, or students point to the actual picture representation of the rectangle with the largest area.

Additional Considerations for Emerging Communicators

1. Modify the word problem to include smaller whole numbers.
    - For example, Alex has 16 yards of fencing to place around a rectangular garden he is making. He wants the area of the garden to be as large as possible. What should the length and width of the garden be?
    - Provide students with picture and/or tactile representations of the important aspects of the problem as it is being read.
    - As information is provided, students organize the information in a graphic organizer, baskets, etc.

2. Provide students with a computer program or premade PowerPoint as used in previous portions of the lesson to explore rectangles with the same perimeter but different areas.

3. Using the computer program or PowerPoint with switch access, students verify that the perimeter is 16 yd. in the modified word problem and count the area units to determine the area.

4. After students have verified the perimeter or determined the area for a particular rectangle, the program should insert the number of units into the correct portion of the chart. Remember, the alternate keyboard or switches used should have picture or tactile representations of area and perimeter so students determine the correct measurement.

See Example: PowerPoint Lesson 1, Slide 8.

5. Students indicate the rectangle with the largest area by choosing rectangle A or B.

Lesson 1: Closure – 5 minutes

A. Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Convert units using standard/known conversion units.
2. Use appropriate known formulas for the area.
3. Solve multistep problems involving one unit of measure.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement*:* Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives key words paired with images. From that information, they share what they have learned based on each of the expectations.
    - For example, the student may grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students refer back to the photographs or models/tactile representations of examples of real-life situations in which these concepts are used to share when they could use these new skills.
    - For example, the student could touch the tactile representations for area and table top to state, "I can use area to determine if an object will fit on my desk."

B. Exit Assessment

1. Students are given a new word problem to solve that includes mixed measurements within the same system.
2. Students work independently to find perimeter and area of rectangles and solve for the situation.
3. Students should use a similar table as that used during practice.

New Problem Example: Josh is designing a display for his Science Fair project. His display must have a perimeter of 120 inches or 10 feet. He found two display boards: board one measures 24 in x 3 ft and board two measures 30 in x 30 in.

- Which display meets the criteria?
- Is it display 1?
- Is it display 2?
- Both?
- None?
- Which display gives Josh the largest display area?

Multiple means of representation: Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models.

Multiple means of engagement: Ensure students are actively involved in their small groups. Present different problems related to students' interests. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Use the same supports as used in the practice section to solve for the given problem.
2. Use the supports described in Lesson Body, Step 4: Converting Units of Measure.

Lesson 1: Resources

The following pages are examples of activities and exercises from Lesson Body, page 8.

Area and Perimeter of Similar Figures:

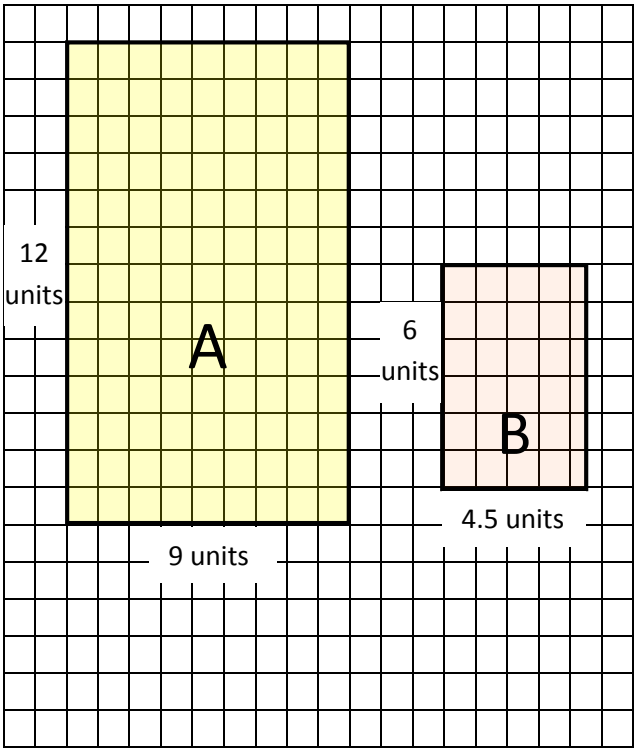


Figure A

A = 12 units x 9 units

A = 108 units^2

P = 12 units +12 units + 9 units + 9 units

P = 42 units

Figure B

A = 6 units x 4.5 units

A = 27 units^2

P = 6 units + 6 units + 4.5 units + 4.5 units

P = 21 units

Compare Area of figures A and B

$\frac{AA}{AB} = \frac{108}{27}$

$\frac{AA}{AB} = \frac{4}{1}$

Compare Perimeter of figures A and B

$\frac{PA}{PB} = \frac{42}{21}$

$\frac{PA}{PB} = \frac{2}{1}$

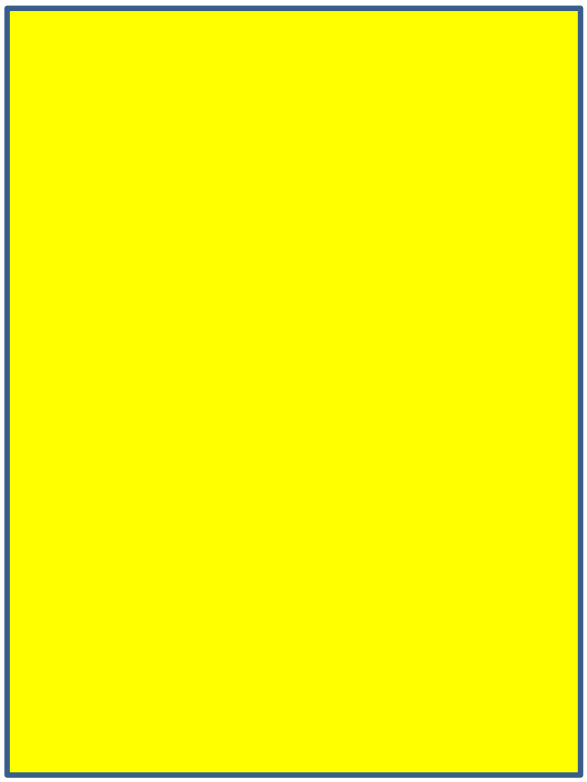
Lesson 1: Resources – continued

The student can use manipulatives (next 2 images) to compare the two figures by laying rectangle B over rectangle A until completely covered to determine how many times bigger area of A is than B. The student can compare perimeter by laying rectangle B over rectangle A to determine how many are needed to create the same length (2), and how many are needed to create the same width (2).

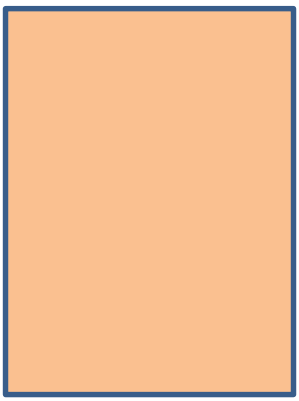
The area of A is _4_ times the area of B. The perimeter of A is _2_ times the perimeter of B.

Tactile representations of similar figures: Cut out figures using construction paper, poster board, card board, sand paper, etc. Representations can also be cut out as templates or frames to lay over grid paper or cut out of transparencies to see grid lines.

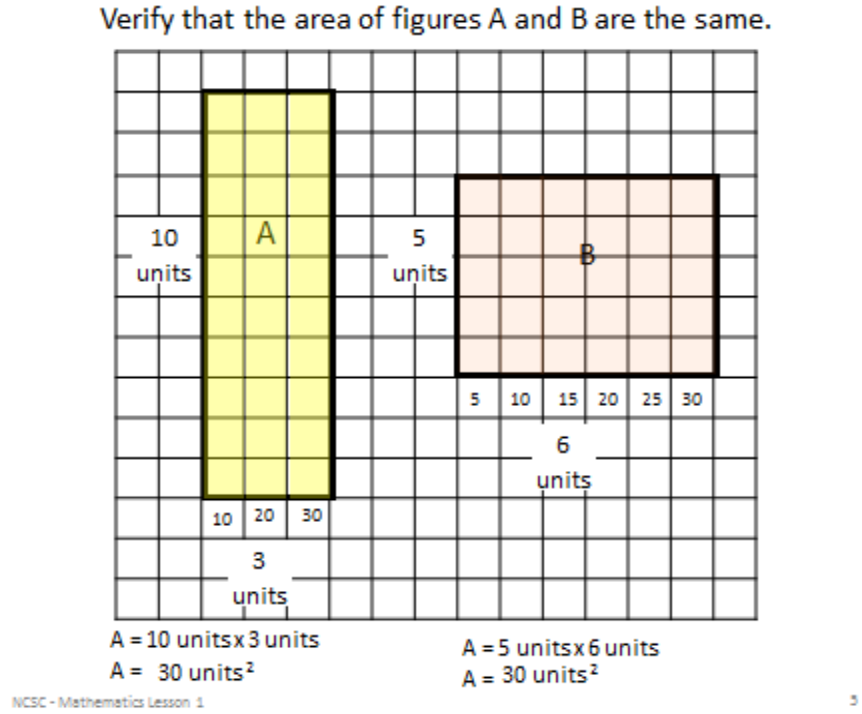
Rectangle A



Rectangle B



Lesson 1: Resources – continued



Is the area of Figure A the same as the area of Figure B?

Figure A:

A= 10 units x 3 units

A= 30 units (squared)

Figure B:

A= 6 units x 5 units

A= 30 units (squared)

Is the perimeter of Figure A the same as the perimeter of Figure B?

Figure A:

P= 10 units + 10 units + 3 units + 3 units

P= 26 units

Figure B:

P= 6 units + 6 units + 5 units + 5 units

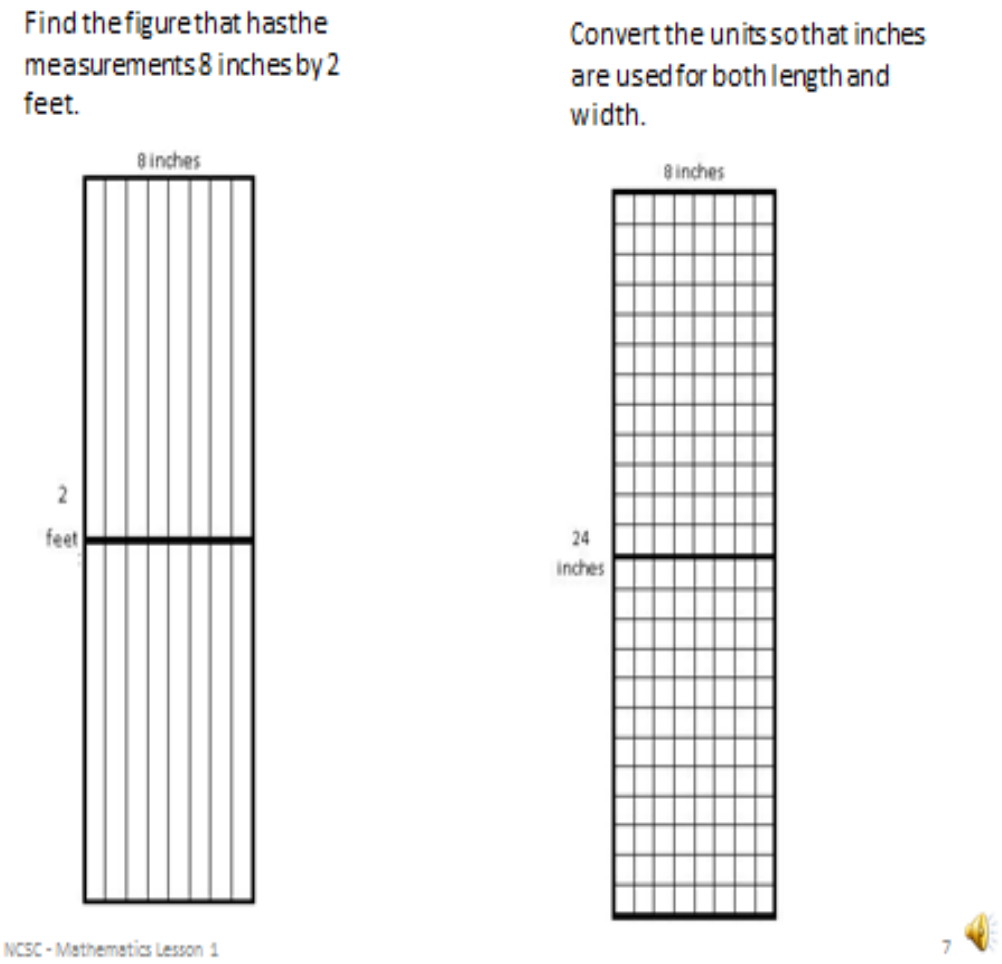
P= 22 units

The perimeter of Figure A and Figure B is:

The same Different

The perimeter of Figure A is <u>more than / less than / the same as</u> the perimeter of Figure B.

Lesson 1: Resources – continued

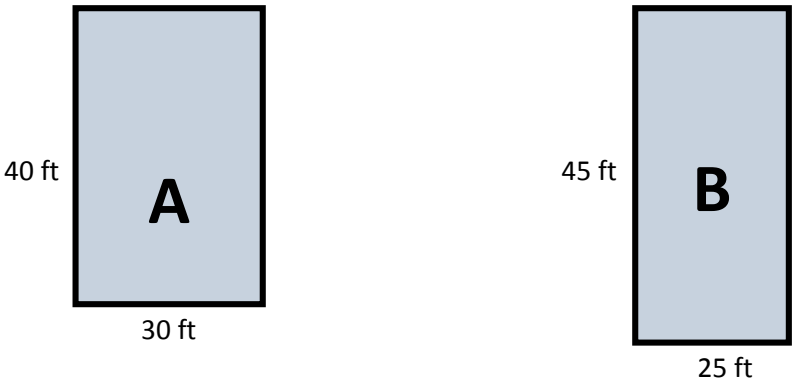


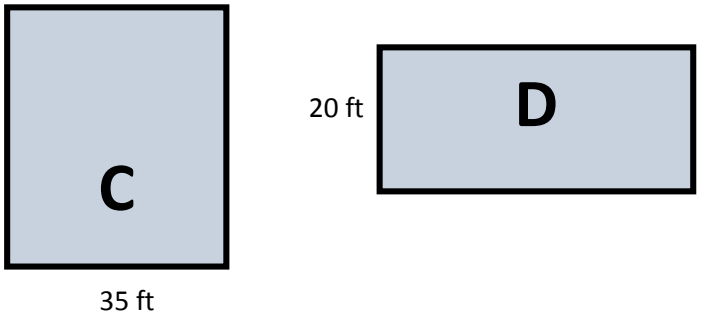
These figures can be represented tactilely with raised, thick, exterior and center lines to represent the height measured in feet. They can also be represented with thinner, raised lines to represent width and height in inches.

Lesson 1: Resources – continued

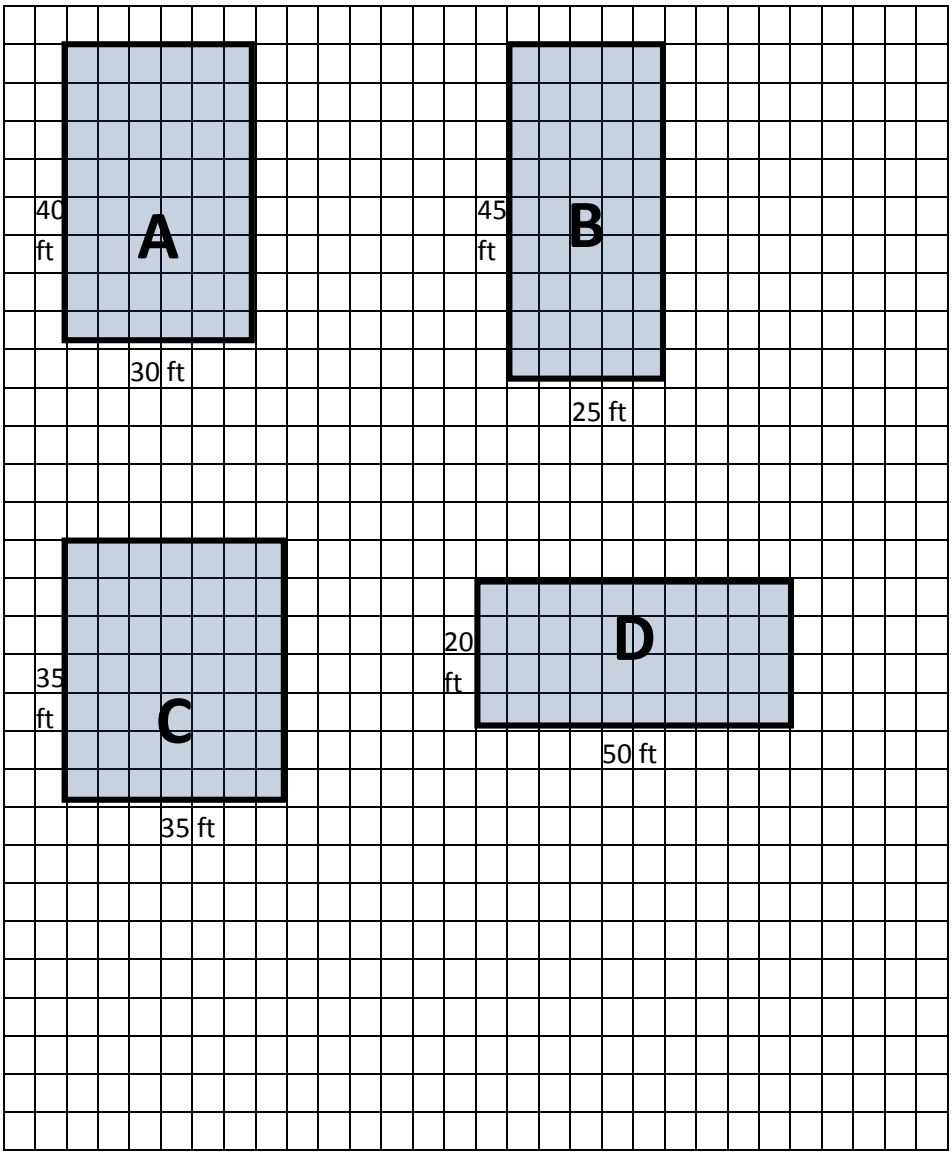
The following pages are examples of activities /exercises from Lesson Practice, page 11.

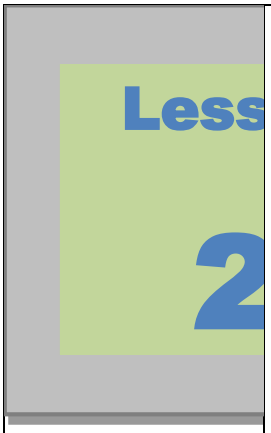
Model	Length	Width	Perimeter	Area
A	**40 ft**	**30 ft**	**140 ft**	**1200 ft^2**
B	**45 ft**	**25 ft**	**140 ft**	
C	**35**	**35**	**140 ft**	
D	**20**	**50**	**140 ft**	





Lesson 1: Resources– continued



	**Content Area:** Mathematics - Measurement
Grade Span: 9 - 10	
Lesson 2 of the Unit	**Approximate Time Needed:** 50 minutes

Objective: Students will make decisions about units and scales that are appropriate for problem solving situations involving mathematics or across disciplines or contexts.
Essential Question(s): 1. How does the area of a figure change when one of its dimensions is doubled, tripled, etc.? 2. How can we use ratios and proportions to determine how the area of figures change when at least one of the dimensions change?

Materials Set Up:
• Provide practice and review worksheets or class discussion about ratio and proportion.
• Activities will be varied and should include individual and group worksheets, text exercises, and hands-on problem solving activities.
 	
Materials Needed:
• Worksheets
• Grid paper
  ○ Grid paper with raised lines
  ○ Grid paper copied on transparencies for light boxes
• Square tiles
• Geoboard
• Pencils
• Overhead projector

- Transparencies
- 12-inch ruler
- Yardstick
- Poster board
- Chalk board, white board or Smart Board

Lesson Vocabulary
Area
Dimensions
Length
Perimeter
Proportion
Ratio
Width

Lesson 2: Introduction – 10 minutes

A. Activate Previous Knowledge

1. Lead discussion on meaning of ratio and proportion.
2. Provide practice worksheets on finding ratios and proportions.

Multiple means of representation: Provide definitions of ratio and proportion. Provide drawings or models of ratios and proportions. Worksheets can be presented with fewer items per page. Worksheets may be printed on various color sheets. Worksheets may be presented using a computer.

Multiple means of expression: Students may express the meaning of ratio and proportion by describing a situation that represents a ratio or proportion. Students may create a model by drawing or using manipulatives to express the meaning of ratio and proportion. Students may use drawings or models to complete worksheets. Students may complete work on a computer.

Multiple means of engagement: Provide examples of ratios and proportions based on students' interests or real life situations in which they would be used.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide picture/tactile/object representation of vocabulary words as they are used throughout the discussion.
    - Provide concrete examples when discussing the meaning.
    - During the discussion, students match a picture representation to the example provided which students use as a reference throughout the lesson.
    - Examples can be provided by using a computer program and adaptive software with a talking word processor.

2. Include picture/tactile/object representations of key words in directions and word problems.

- Provide graphic or manipulative representations of proportional relationships or use a computer program with alternate keyboard or switch access and talking word processor of proportional relationships.

Lesson 2: Introduction – continued

B. Establish Goals/Objectives for the Lesson

Inform students that in this lesson the will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts.

Explain that they will:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/tactile representations.

2. Provide the key words in the lesson objectives paired with images or tactile representations, record into mathematics journals. Students may use an electronic picture writer to record the lesson objectives.

3. Provide students with visual and/or tactile representations of situations when these concepts are used.

Lesson 2: Body – 20 minutes

Direct Instruction and/or Facilitation of the Lesson

1. Students draw rectangles of various dimensions on grid paper or form on Geoboards or with square tiles and to determine the area of each rectangle. Discuss why the area of the rectangles are the same or different.

2. Students draw or form a rectangle A with given dimensions (e.g., 10"x5"). Then have students draw or form rectangle B by doubling the length of first one side and both sides of rectangle A. Find the ratio of the area of rectangle A to the area of rectangle B.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter, or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc. to complete exercises.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Students should determine the area of various given rectangles using the same strategies and supports as used in lesson 1.

2. Draw a model of rectangle A with given dimensions (e.g., 5" x 4") or provide a manipulative model or virtual template of rectangle A. Students place the model on grid paper and verify the length and width by counting the units along the length of the figure and the width of the figure. Students determine the area by counting the units within the figure. Students should keep a record of the length, width, and area of rectangle A.

Rectangle	Length	Width	Area
A	5 in	4 in	20 in^2
B			
C			
D			

3. Once students have determined the area of rectangle A, provide the rule and/or model for setting up a ratio of area to width.
    - The letters representing length, width, and area can be paired with a picture representation of each measurement.

See Example: PowerPoint Lesson 2, Slide 1.

4. The ratio of area to width for rectangle A is $\frac{A}{W}$.

5. Provide a template of rectangle B with given dimensions as the width twice as wide as rectangle A. Students should keep a record of the length and width of rectangle B (e.g., 5" x 8").

6. Once students have recorded the length and width of rectangle B, provide the rule and/or model for setting up the ratios of rectangles A and B to determine the area of B.
    - The letters representing length, width, and area can be paired with a picture representation of each measurement.

A	B

$$\frac{A}{W} = \frac{A}{W}$$

See Example: PowerPoint Lesson 2, Slide 2.

7. Using the information from the chart, input the numbers into the proportions using the proportions for area and width.

A	B

$$\frac{20in^2}{4in} = \frac{A}{8in}$$

8. Using the templates, students demonstrate that it takes 2 of rectangle B to make the same width of rectangle A.
    - Students should conclude that the width of rectangle B is 2 times the width of rectangle A.
    - Allow students to use a calculator to verify that multiplying the width of A by 2 will give students the width of B (4 x 2=8)

A	B

$$\frac{20in^2}{4in\ \boldsymbol{x2}} = \frac{A}{8in}$$

9. Explain: If you multiply the bottom number by 2, you must multiply the top number by 2.
    - Allow students to use a calculator to determine the area of rectangle B.

A	B

$$\frac{20in^2\ \boldsymbol{x2}}{4in\ x2} = \frac{A}{8in}$$

$\frac{20in^2\ x2}{4in\ x2} = \frac{40in^2}{8in}$

10. Students place the template on grid paper to determine the area by counting the units within the figure.

See Example: PowerPoint Lesson 2, Slide 3.

11. Repeat by providing students with a template for rectangle C (10" x 4"), the length of which is double that of rectangle A.
    - Use the proportional formulas for area and length.

Lesson 2: Practice – 10 minutes

Repeat the exercise by tripling the dimensions and/or by reducing the dimensions by one-half.

Multiple means of representation: Use models and/or drawings during large group instruction. Allow students to have a copy of a drawing or a model at their desks.

Multiple means of expression: Provide a list of formulas to determine area and perimeter, or provide options for using manipulatives and/or computer models.

Multiple means of engagement: Allow students to use paper/pencil, manipulatives, computer, etc., to complete exercises.

Additional Considerations for Emerging Readers and Emerging Communicators

- Use the same supports as used throughout the lesson body.

Lesson 2: Closure – 10 minutes

A. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.

Multiple means of representation: Along with posted lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students share what they have learned in different formats: through writing, drawing, creative expression, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or in their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives' key words paired with images or tactile/objects representations so they can share what they have learned based on each of the expectations.
    - For example, a student may touch the tactile cues for ratio and area to state, "I have learned how to compare areas of similar figures by using ratios."

3. Students refer back to the photographs or tactile representations of examples of real-life situations in which these concepts are used to share when they could use these new skills.

B. Exit Assessment

1. Generalize the effect on the area of the rectangle when one and/or both dimensions are multiplied by a factor of "n."

2. Students review their work from the lesson and conclude that when only one dimension is changed, the area is affected by the same amount of change, but if both dimensions are changed, the area is affected by the change of width times the change of length.

Multiple means of representation: Allow students to refer back to their work samples, models, drawing, notes, etc.

Multiple means of expression: Students share what they have learned in different formats: writing, drawing, creative expression, etc.

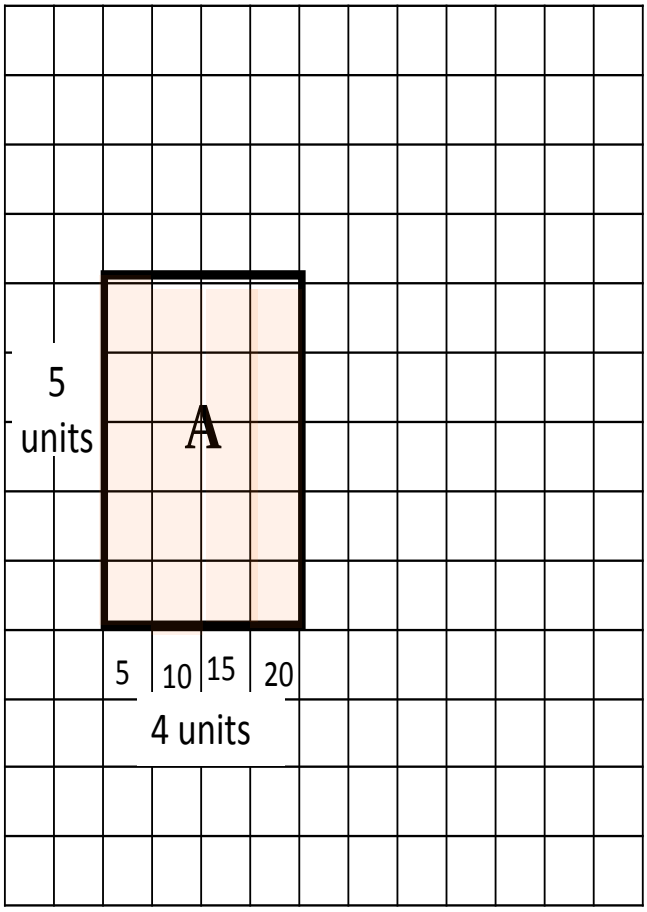
Multiple means of engagement: Allow students to review work independently or to review with a partner or small group.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Students show the effect on the area of the rectangle when one and/or both dimensions are multiplied by a factor of "n" by drawing models or using manipulatives to demonstrate how many of one figure is needed to cover the other.

2. Use ratios and proportions to show the relationship between the areas of the original rectangle and the new rectangle that was created by changing dimensions by a factor of "n."

Lesson 2: Resources



What is the ratio of Area to Width of rectangle A?

$$\frac{A}{W} = \frac{20\ u^2}{4\ u}$$

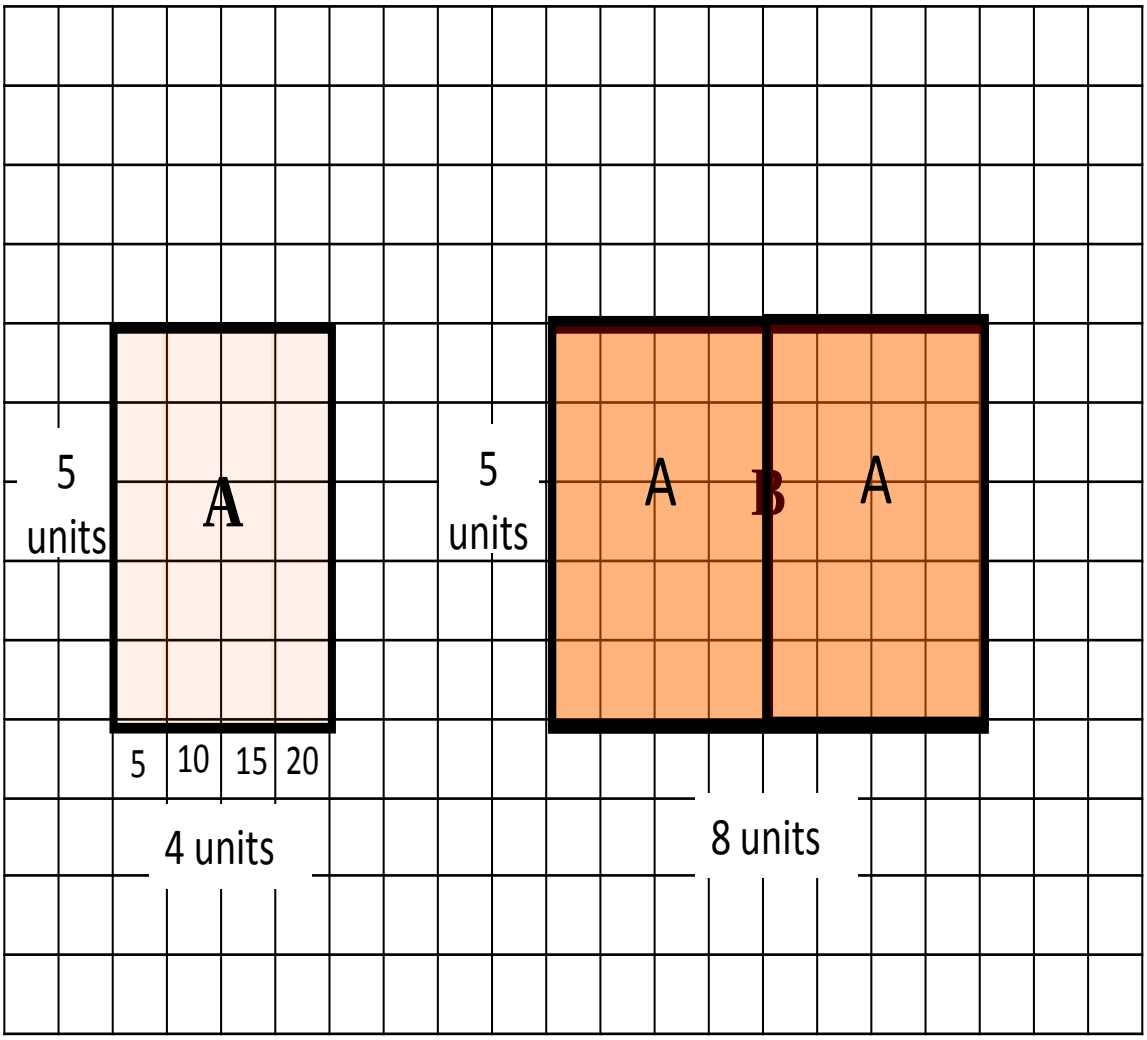
Determine the area of rectangle A.

A $5u \times 4u = 20\ u^2$

NCSC – Mathematics Lesson 2

1

Lesson 2: Resources– continued



What happens to the area of rectangle A if we double the width?

If the ratio of Area to Width of rectangle A is $\frac{A}{W} = \frac{20\ u^2}{4\ u}$

Use the ratio $\boxed{A}\ \frac{A}{W} = \boxed{B}\ \frac{A}{W}$ to figure the Area of rectangle B?

NCSC – Mathematics Lesson 2

2

Grade Span: 9 - 10	**Content Area:** Mathematics – Measurement Investigating Measurement in the Real World
Lesson 3 of the Unit	**Approximate Time Needed:** 90 minutes

Objective: Students will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts.

Essential Question: How can we use proportion to convert measurements from one unit to another in the same system?

Materials Set Up:

- Provide practice and review worksheets or class discussion about ratio and proportion.
- Activities will be varied and should include individual and group worksheets, text exercises, and hands-on problem solving activities.

Materials Needed:

- Worksheets
- Grid paper
- Grid paper with raised lines
- Grid paper copied on transparencies for light boxes
- Square tiles
- Geoboard
- Pencils
- Overhead projector
- Transparencies
- 12-inch ruler
- Yardstick
- Poster board
- Chalk board, white board, or Smart Board

Lesson Vocabulary:
Area
Centimeter
Conversion
Foot

Inch Length Meter Proportion Ratio Width Yards

Lesson 3: Introduction – 15 minutes

A. Activate Previous Knowledge

1. Lead a discussion on the meaning of ratio and proportion.

2. Provide a practice worksheet(s) on finding ratios and proportions.

Multiple means of representation: Present illustrations or models of ratio and proportions during discussions.

Multiple means of expression: Allow students to use paper and pencil, models, computers, etc. to practice the concepts of ratios and proportions.

Multiple means of engagement: Present real life uses for ratios and proportions related to students' interests.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide picture and/or tactile representations of ratio and proportion as well as concrete examples throughout the discussion.
    - Examples can be provided using a computer program and adaptive software with a talking word processor.
    - Refer to ratios and examples used in Lesson 2.

2. Include picture representations of key words in directions and word problems.
    - Provide graphic or manipulative representations of proportional relationships or use a computer program with a switch or alternate keyboard access and talking word processor.

B. Establish Goals/Objectives for the Lesson

Inform students that they will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts and:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.

6. Solve problems requiring calculations that involve different units of measure within a measurement system.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/tactile representations.

2. Provide the key words in the lesson objectives paired with images/symbols/tactile representations, record into mathematics journals or use an electronic picture writer to record the lesson objectives.

3. Provide students with photographs/objects/tactile representations of examples of situations when these concepts are used.

Lesson 3: Body – 30 minutes

Direct Instruction and/or Facilitation of the Lesson

1. Lead the discussion on the types of customary units used to measure length: inches, feet, and yards.

2. List the common conversions: 12 in = 1 ft; 3 ft = 1 yd; 36 in = 1 yd.

3. Show that when converting a larger unit to a smaller unit, we multiply (e.g., 4 ft = 4 x 12 = 48 in) or set up and solve a proportion:
   a. $\frac{1\ ft}{12\ in} = \frac{4\ ft}{x\ in}$

   b. $\frac{1\ yd}{36\ in} = \frac{2\ yd}{x\ in}$

   c. $\frac{1\ yd}{3\ ft} = \frac{2\ yd}{x\ ft}$

4. Show that when converting a smaller unit to a larger unit, we divide: (e.g., 6 in = 6/12 =1/2 ft):
   a. Convert feet to yards: Number of feet ÷ 3= # of yds
      e.g., 2 ft ÷ 3 = 2/3 yds
   b. Convert inches to yards: Number of inches ÷ 36= # of yds
      20 in ÷ 36 = 20/36 or 5/9 yd
   c. Use ratio and proportion:
      $\frac{1\ ft}{12\ in} = \frac{x\ ft}{6\ in}$

      $\frac{3\ ft}{1\ yd} = \frac{2\ ft}{x\ yd}$

5. Convert units of measure for area:
   a. Show using grid paper, tiles, or Geoboard that a square measuring 12 in on a side is the same size as a square measuring 1 ft on a side.
   b. The area of a 12-in by 12-in square = 144 sq. in. or in^2
   c. The area of a 1 ft square = 1 sq ft or ft^2
   d. Therefore, 144 sq in = 1 sq ft or ft^2
   e. In like manner, show 9 sq ft = 1 sq yd or yd^2

6. Discuss how measurements and area can be used in real world situations.
   a. For example, area of floor for tile or carpet, length and width of pictures for frames, area of table tops for tile or tablecloth, length and width of windows for curtains, etc.
   b. Provide students an opportunity to communicate ideas with the class.

Multiple means of representation: Use models and/or drawings on grid paper during large group instruction. Allow students to have a copy of a drawing or a model at their desks. Provide examples of measuring tools. Provide a list of conversion formulas to convert between measurements within the same system.

Multiple means of expression: Allow students to use paper/pencil, manipulatives, computer, etc., to complete exercises.

Multiple means of engagement: Allow students to brainstorm ideas by writing descriptions of examples, drawing examples, acting out examples, etc.

Additional Considerations for Emerging Readers

1. Provide picture representations of inch, foot, and yard as well as concrete examples throughout discussion.

2. Remind students that when the measurements increase, use multiplication.
   - Remind students how to make sure the ratio remains balanced: Whatever is multiplied on the top must be multiplied on the bottom.
   - Refer back to supports used in the introduction.
   - $\frac{1\ ft}{12\ in}$ x 4 $= \frac{4\ ft}{x\ in}$

3. Remind students that when the measurements decrease, use division.
   - Remind students how to make sure the ratio remains balanced: Whatever is divided on the top must be divided on the bottom.
   - Refer back to supports used in the introduction.

   $\frac{1\ ft}{12\ in}$ ÷ 2 $= \frac{x\ ft}{6\ in}$

4. Students measure and draw a square that is 12 inches by 12 inches.
   - Using the formula for area, students determine the area in square inches.
   - Students measure the square again using the measurement of foot.
   - Students should see that it is 1ft x 1ft and equals 1 ft^2.
   - Students should conclude that $144in^2 = 1ft^2$.

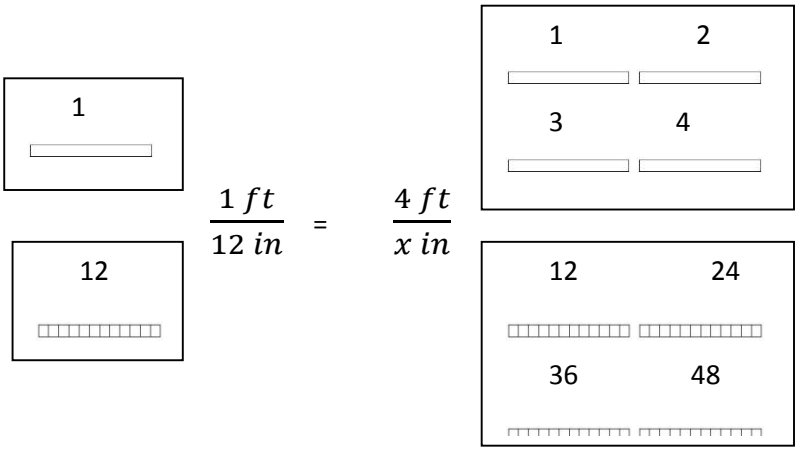
5. Students measure and create a square that is 3 ft x 3ft by taping it out on the floor and determine the area.
   - Students measure the same square using a yardstick.
   - Students should determine the square is also 1yd x 1 yd and equals 1 yd^2.
   - Therefore, $9ft^2$ and $1yd^2$ are equal.

See Example: Lesson 3 Conversions.

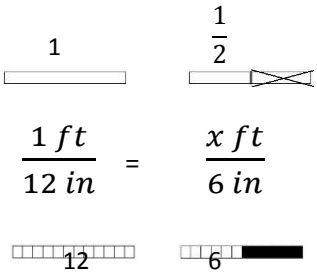
Additional Considerations for Emerging Communicators

1. Provide picture and tactile/object representations of inch, foot, and yard as well as concrete examples of measurement tools for the different units throughout discussion.

2. Provide students with concrete examples of the ratios when increasing measurements.



3. Provide students with concrete examples of the ratios when converting to smaller units.



See Resources: Lesson 3 tools to use, pages 47- 49.

Lesson 3: Practice – 30 minutes

1. Model a problem with the class that involves making decisions about units and scales, and determine various ways to solve it.
    a. Problem 1: A floor is 9 ft wide and 12 ft long. How many tiles (12" on a side) are needed to completely cover the floor?
    b. Draw a rectangle to represent dimensions 9 ft by 12 ft or make a scale drawing of it.

2. Model a second problem for students.
    a. Problem 2: A floor is 9 ft wide and 12 ft long. How many tiles (18" on a side) are needed to completely cover the floor?
    b. Students recommend whether to convert the floor plan to inches or the tiles to feet.
    c. If students recommend converting 9 ft and 12 ft to inches, then find the area of the rectangle in square inches (108in x 144in) and divide by the area of a tile (12in x 12in).

    For example:
    - 9 x12in = 108 in 12 x 12in = 144 inches
    - Area of the floor in inches: 108 in x 144 in = 15,522 in^2
    - Area of the tile in inches: 18 in x 18 in = 324 in^2
    - 1552 in^2 ÷ 324 in^2 = 48 in^2, so 48 tiles are needed

Multiple means of representation: Allow students to have a written copy of the problem, drawn models of the situation, and/or conversion formulas as needed/requested.

Multiple means of expression: Students may draw or use manipulatives to model solutions or use the computer.

Multiple means of engagement: Create situations that include areas of interest to students.

Additional Considerations for Emerging Readers

1. Provide the written problem to include picture representations of relevant words so students can follow along as the problem is introduced.

- Provide students a scale drawing of the rectangle (floor) on grid paper measuring 9 units by 12 units with each unit representing a foot.
    - Label the rectangle "floor" using word and picture representation.
    - Each square in the grid represents 1 inch in length and 1 inch in width.
    - Highlight around 12 in x 12 in to represent 1 ft x 1 ft.
    - Provide students with paper or object squares that equal the size of a square foot on the graph paper.

- Use these to represent the 12" tiles.
- Allow students to use the conversion chart from previous lessons to determine how to best convert the units.
- Convert the floor to inches or convert the tiles to feet.

- Once students have made the conversion, determine the area of the tile using the equation length x width.
  - Students should discover that the area of the tile is $1ft^2$.
  - Use the ratio $\frac{1\ tile}{1\ ft^2}$ to determine how many tiles are needed to cover the area of the floor: $\frac{1\ tile}{1\ ft^2} = \frac{?\ tiles}{108\ ft^2}$
- Students use a calculator to determine and solve the proportions.
  - Or students determine how many tiles are needed to complete the length of one side of the floor by placing a manipulative tile on the floor plan and count 9 tiles needed and repeat for the width, counting 12 tiles.
  - Students multiply 9x12 to determine the number of tiles needed to complete the floor.

2. Provide the written problem to students to include picture representations of relevant words so students can follow along as the problem is introduced.

- Be sure students have the picture representations of inches, feet, tile, and floor so they can give an opinion as to which rectangle should be converted to which unit of measure.
- Students should still have the scale drawing of the rectangle (floor) on grid paper measuring 9 units by 12 units with each unit representing a foot and the rectangle labeled "floor" using word and picture representations.
- Provide students with paper or object squares that represent 18" scaled to one and a half the size of the grid paper unit.
- Given a model of the multiplication problem for converting feet to inches (___ft x 12 inches) and a calculator, students convert the length and the width of the floor from feet to inches.
  - 9 x 12in = 108 in
  - 12 x 12in = 144 inches
- Using the formula for area, students use the calculator to determine the area of the floor (108 in x 144 in) and the area of the tile (18 in x 18 in).

- Students create a ratio of the area of the floor to the area of the tile then divide: $\frac{15522\ in^2}{324\ in^2} = 48$.

Lesson 3: Practice – continued

Additional Considerations for Emerging Communicators

1. Provide students with relevant picture or tactile/object representations of relevant words/concepts as the problem is introduced.
    - Use real or replicas of 12-inch tiles and create a 9 ft by 12 ft rectangle on the floor using colored tape or a computer program to model the floor plan.
    - Using the foldable ruler, review that 12 inches is the same as one foot.
    - Convert scale representation of tiles from 12 inches to one foot.
    - Place manipulative tiles down the length of one side of the taped floor plan and count how many tiles are needed to cover the length of that side.
    - Repeat for one side the width of the floor.
    - Multiply the two numbers to determine the number of tiles needed to cover the floor or use a computer program to input the tiles and input the numbers into a multiplication problem to solve the problem.

2. Provide students with picture or tactile/object representations of relevant words/concepts as the problem is introduced.
    - Use real or replicas of 12-inch tiles and create a 9 ft by 12 ft rectangle on the floor using colored tape or a computer program to model the floor plan.
    - Using one foot rulers, show that it takes 9 rulers to cover the length of the floor.
    - Using the foldable ruler, review that there are 12 inches in a foot.
    - Convert the length and width of the floor from feet to inches by inputting the correct number to complete the multiplication problem (___ft x 12 inches) using a calculator or computer program.
    - Place manipulative tiles down the length of one side of the taped floor plan and count how many tiles are needed to cover the length of that side.
    - Repeat for one side the width of the floor.
    - Multiply the two numbers to determine the number of tiles needed to cover the floor or use a computer program to input the tiles and input the numbers into a multiplication problem to solve the problem.

Important Note for Communicators Considered Pre Symbolic: The number load may need to be reduced.

Lesson 3: Closure – 15 minutes

A. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations involving mathematics or across disciplines or contexts, and:

1. Set up and solve proportions.
2. Convert units of measurement using standard/known conversions.
3. Recognize when to multiply and when to divide in converting measurements.
4. Use ratio and proportion to convert measurements.
5. Use appropriate known formulas for area.
6. Solve problems requiring calculations that involve different units of measure within a measurement system.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, students refer to the lesson objectives they recorded or collected in their mathematics journals or in their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives key words paired with images. From that information they share what they have learned based on each of the expectations.
    - For example, students may grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students should refer back to the photographs and/or tactile representations of examples of real-life situations when these concepts are used to share when they could use these new skills.
    - For example, students could touch the tactile representations for area and tabletop to state, "I can use area to determine if an object will fit on my desk."

Lesson 3: Closure – continued

B. Exit Assessment

1. Students solve a third problem using models and paper and pencil.
    - For Example, How many 6 in square tiles are needed to cover a 3 ft. by 5 ½ ft. counter top?
        - Discuss the results showing more than one strategy.

Multiple means of representation: Allow students to have a written copy of the problem, drawn models of the situation, and/or conversion formulas as needed/requested.

Multiple means of expression: Students may draw or use manipulatives to model solutions or use the computer.

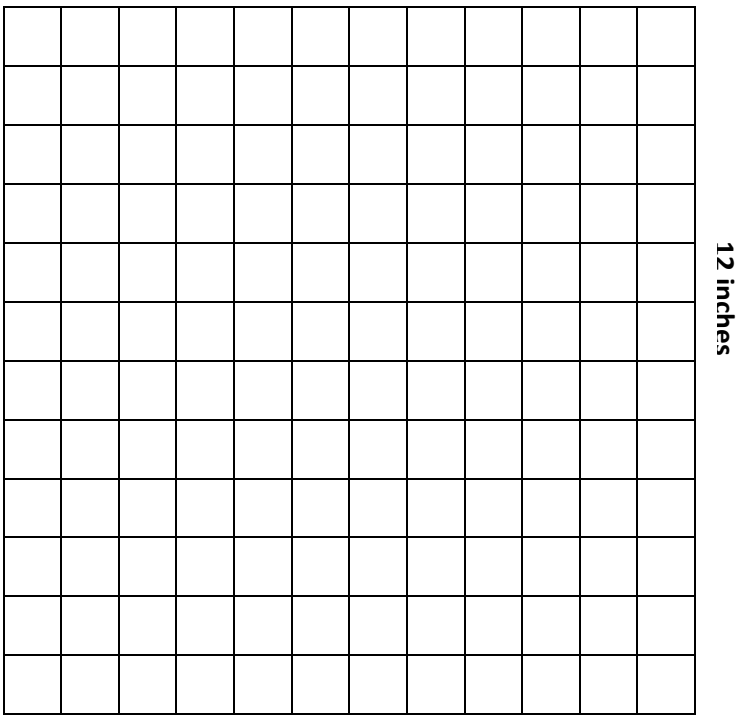
Multiple means of engagement: Create situations that include areas of interest to students.

Additional Considerations for Emerging Readers and Emerging Communicators

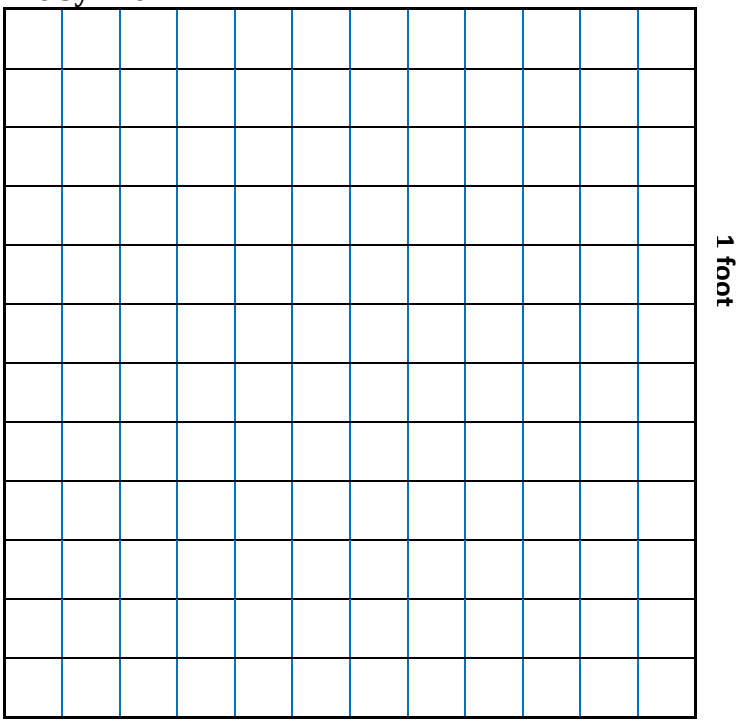
- Use the same supports as used in the practice section.

Lesson 3: Resources

12 in x 12 in



1 ft by 1 ft



Lesson 3: Resources– continued

9 ft by 9 ft

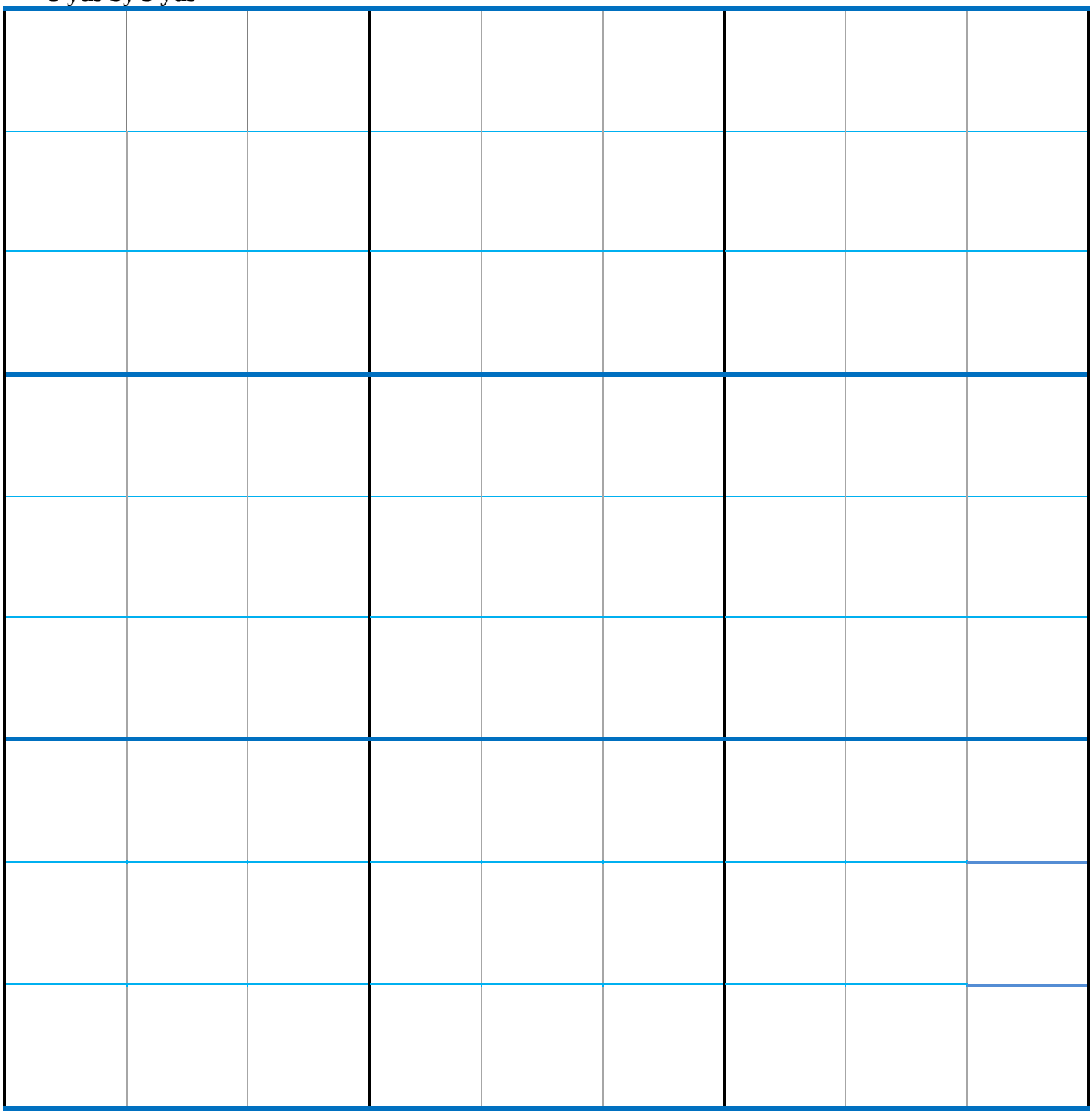
9 feet

9 feet

Lesson 3: Resources– continued

3 yds by 3 yds

3 yards



Grade Span: 9 - 10	**Content Area:** Math - Geometry
Lesson 4 of the Unit	**Approximate Time Needed:** 55 minutes

Objective: Students will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts.

Essential Questions:

1. What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?
2. How does changing the sides of a square affect the area?

Materials Set Up:

- Set up problem and question on overhead/whiteboard.
- Mark off floor in varying size squares up to 25 square feet. (e.g., 1'x1', 2'x2', 3'x3', 4'x4', 5'x5') to demonstrate size needed for dancing.
- Create or select manipulatives to represent area and people.

Materials Needed:

- Calculator
- Pencils, paper, graph paper, masking tape
- Manipulatives
- CD player with various types of music
- Blank table with columns for name and dance space
- Measuring tape, yardstick, ruler

Lesson Vocabulary

Area
Length, Width
Ratio
Unit of Measure
Unit Rate

Lesson 4: Introduction – 10 minutes

A. Activate Previous Knowledge

1. Remind students that they have been working on the concepts of perimeter and area of shapes of different sizes.

2. Review ratios.

3. Discuss how they can use this knowledge to solve problems they may encounter in the real world.

4. Present the following idea/problem:
   - Introduce the idea of planning a dance party in the classroom.
   - How many people would you expect to attend?
   - How large will the dance floor need to be?

5. Students use previous knowledge to brainstorm ways to solve this problem.

Multiple means of representation: Present real life problems using drawings, models, and video representations of people dancing on a dance floor.

Multiple means of expression: Allow students to present ideas for problem solving using computer models, demonstrations, visuals, etc. Record problem solving ideas in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Use student-chosen dance styles and music when presenting problem. Allow students to work individually or in small groups based on learning style.

Additional Considerations for Emerging Readers and Emerging Communicators

1. During the review, be sure students have graphic and/or tactile representations of relevant vocabulary (area, perimeter, length, width) as well as related materials/drawings/objects representations from previous lessons.

2. Provide examples through pictures, tactile cues and/or videos of couples on various dance floors that are very crowded, average, and very empty, etc.
   - Point out how much space people have to dance.
   - Ask guiding questions:
     - "Do the dancers have enough space?"
     - "Is there room for more people?" etc.
   - Students attempt to add more people to an object representation or virtual representation of the dance floor.
   - Create a list of possible dance styles by using words paired with pictures.

3. Students use their math journals to refer to previous strategies used for solving problems. Ensure students have a way to contribute to the brainstorming.

Lesson 4: Introduction – continued

B. Establish Goals/Objectives for the Lesson

Inform students that they will make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Identify, quantify, and compare the attributes of the objects, situations, and/or events that need to be measured to solve the problem/situation.
2. Use appropriate units of measure to identify, quantify, and compare objects, situations, and/or events to solve a real world problem.
3. Convert units when necessary.
4. Represent data in various forms

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/tactile representations (i.e., units of measure, inch, foot, yard, centimeter, and meter).

2. Provide the key words in the lesson objective paired with symbols/images/tactile representations to paste into their mathematics journals or collect in a mathematics basket or bag. Students may use an electronic picture writer to record the lesson objectives.

3. Provide students with photographs, models, or tactile representations of examples of situations when these concepts are used.

Lesson 4: Body – 15 minutes

Direct Instruction and/or Facilitation of the Lesson

Whole Group Discussion:

1. Review students' ideas on how to solve the dance floor problem. Pull out relevant ideas to try making sure the following are included:
    a. Measure a square/rectangular area in the classroom that could be used as a dance floor.
    b. Choose the best unit of measurement to measure the space.
    c. Explore strategies for determining how much space each person needs to dance depending on type of music.
        - What is the best unit of measurement to use?
        - Introduce unit rate, square feet per couple.

2. In small groups, students list at least three types of dance/music they will model and measure to determine how much floor space is needed per person.
    - Allow students to volunteer to dance.
    - Students demonstrate different styles of dancing, slow or fast.
    - Students who did not want to dance should measure and record the dance space needed per person and per style of dance in a table using appropriate unit of measurement, square footage.
    - Be sure the amount of dance space needed for students in wheelchairs is considered.
    - Display the information in the table at the front of the classroom.

Dance Style	Space Needed for a Person		
	Length	Width	Area
Slow dance	*2 ft*	*2ft*	*$4ft^2$*

Multiple means of representation: Allow students to refer to their brainstorming notes during discussion. When discussing unit rate, provide familiar examples (e.g., miles per hour). Provide students with a copy of the word problem and the table above. Have drawings and manipulatives available for students to use. Provide options for demonstrating different dance styles (e.g. volunteers demonstrate, bring dancers into the classroom to demonstrate, watch video demonstrations, etc.)

Multiple means of expression: Allow students to solve the problem using formulas and/or models and record information into the tables using various formats paper and pencil, computer, etc.

Multiple means of engagement: Ensure all students are actively involved in their small groups. Use music and dance styles related to students' interests. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers

1. Introduce the concept of unit rate as area per person or the amount of space needed per person to have enough room to dance.
    - Provide a list of brainstorming ideas for solving the problem.
    - Include picture and/or tactile representations of different dance styles as needed.

2. Be sure all students have a job in their small groups that relates to the mathematical concept of measuring.
    - When students are measuring, be sure they have the needed supports for reading the ruler/yardstick to the nearest foot and inch.
    - Provide students with a copy of the table as well as word/picture representations of the different dance styles that can be used to complete column one.

Additional Considerations for Emerging Communicators

1. Introduce the concept of unit rate as area per person or the amount of space needed per person to have enough room to dance.

2. Represent some of the brainstorming ideas they came up with in the introductory lesson using tactile and object representations to review the ideas with the class.
    - Provide some new ideas in tactile representation that students share with the class.

3. Be sure all students have a job in their small groups that relates to the mathematical concept of measuring.
    - When students are measuring, they keep track of the number of feet by working with a peer.
        - Each time the peer lays the ruler and measures a foot, the other student places a tactile representation of a foot long in a basket for length and a tactile representation of a foot wide for width.
        - When finished, students count how many feet were collected/measured for length and width.

4. Observe how students communicate within their groups.
    - Ensure they have a means for sharing ideas and gaining peers' attention.
    - If not, allow opportunities to practice within the group.
    - Be sure activities are engaging to encourage communication.

5. Provide students with a copy of the table as well as word/picture/tactile representations of the different dance styles that can be used to complete column one.
    - If students are completing the table, they place a tactile representation for each dance style in column one.
    - Record the number of feet for length and width by tallying each time a foot is measured.

Lesson 4: Practice – 15 minute

1. Break into small groups to solve the problem.
    - Pose the problem: Your class is having a party and wants a dance floor. The biggest dance floor in the classroom is 8ft x 8ft.
    - Based on each dancer's estimation of the dance space needed for one person preforming one style of dance, how many people can dance at one time on the dance floor?
    - Calculate each style separately by using unit rate.

Dance Style	Space Needed for One Person			Number of People Who Can Dance at One Time
	Length	Width	Area	
Slow dance	*2 ft*	*2ft*	*$4ft^2$*	*16*

- Based on the information in the table, what style of music would you play?
- Explain why.

2. Bring the whole group back together.
    a. Fill in the table to indicate the number of dancers who can dance at one time based on the style of dance.
        - Unit rate of couple per square feet needed based on different dance styles.
    b. Discuss the style of music the class would choose.

Multiple means of representation: Provide students with a copy of the word problem, a template of the formulas for the unit rate/ratios and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using the formulas, drawings, computer graphics, and/or models, etc. Record the number of people into the tables using various formats: paper and pencil, Smart Board, computer, etc.

Multiple means of engagement: Ensure all students are actively involved in their small groups, and use music and dance styles related to students' interests. Use questioning to encourage students to explain their strategies.

Lesson 4: Practice – continued

Additional Considerations for Emerging Readers

1. Present students with the problem written with words paired with picture and/or object representations of the most salient vocabulary from the problem.
    - Determine the area of the dance floor, using previous learned strategies (e.g., 8ft x 8ft = $64ft^2$).
    - Using information from the table, use the unit rate and equivalent ratios to determine how many people can dance on the floor at one time:
        - Slow dance $\frac{1\ person}{4\ ft^2} = \frac{x\ person}{64\ ft^2}$
    - Repeat for each style of music.

2. Individuals from groups share their results to complete the class table. Students should have their own copies of the table for reference. Provide picture/number representations for students to use to communicate results if needed.

Additional Considerations for Emerging Communicators

1. Present the problem to students written with words paired with pictures or object representations.
    - Have a section of the classroom floor measured out and taped in an 8ft x 8ft square.
    - Students move around the perimeter of the dance floor and within the area.
    - If the floor has one foot square tiles, students skip count the tiles by 8, hitting a preprogrammed switch or placing a representation of each long foot (1x8) in a basket and determine the total of 64.
    - Students stop hitting the switch at the end of the length or stop adding a long foot to indicate understanding/performance of area.
    - Repeat process for each style of music.
    - Students should collect a representation of their group's results as the group collects information on each style.

2. Individuals from groups share the results to complete the class table.
    - Students should use their own copies of the group results, which they collected in their mode of communication, to contribute to the whole class table.

Lesson 4: Closure – 15 minutes

A. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Identify, quantify, and compare the attributes of the objects, situations, and/or events that need to be measured to solve the problem/situation.
2. Use appropriate units of measure to identify, quantify, and compare objects, situations, and/or events to solve a real world problem.
3. Convert units when necessary.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives key words paired with images. From that information, they share what they have learned based on each of the expectations.
    - For example, students grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students should also refer back to the photographs or tactile representations of real-life situations in which these concepts are used to share when they could use these new skills.
    - For example, students touch the tactile representations for area and tabletop to state, "I can use area to determine if an object will fit on my desk."

Lesson 4: Closure – continued

B. Exit Assessment

1. Tell students: "This will be your ticket out the door":
    - If you have a party at your house and have a 10ft x 10ft dance floor, determine how many people could dance at the same.
    - Use the information from the table created during the lesson body.

2. Observe how students solve the problems.
    - Take anecdotal notes or assess students using a rubric.
    - The situation/problem dictates the type of measurement to use.
        - Did students determine the area of the dance floor correctly?
        - Could students use the area to compute the answer to the question?
        - Did students use the rate and equivalent ratios to determine how many people could dance a certain style at one time?

3. Students return to whole group.
    - Small groups present their solutions to the class and explain their process for determining their answers.

Multiple means of representation: Provide students with a copy of the word problem, a template of the formulas for the unit rate/ratios and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas, drawings, computer graphics, and/or models, etc. Record the number of dancers into the tables using various formats: paper and pencil, Smart Board, computer, etc.

Multiple means of engagement: Ensure all students are actively involved in their small groups, and use music and dance styles related to students' interests. As you observe group work, use questioning to encourage students to explain their strategies.

Lesson 4: Closure – continued

Additional Considerations for Emerging Readers
1. Present students with the problem written with words paired with picture symbols of the most salient vocabulary from the problem. • Determine the area of the dance floor, using previous learned strategies (e.g., 8ft x 8ft = $64ft^2$). • Using information from the table, use the unit rate and equivalent ratios to determine how many people can dance on the floor at one time: ○ Slow dance $\frac{1\ person}{4\ ft^2} = \frac{x\ person}{64\ ft^2}$ • Repeat for each style of music. 2. Provide students with a copy of a modified rubric and review expectations. 3. Allow students to use all supports provided throughout the unit lesson so far. 4. Students should have their own copies of the table for reference. 5. Provide picture/number representations for students to use to communicate results if needed.
Additional Considerations for Emerging Communicators
1. Present students with the problem written with words paired with pictures or an object representation of the problem. • Have a section of the classroom floor measured out and taped in an 8ft x 8 ft square. • Students move around the perimeter of the dance floor and within the area. • If the floor has one foot square tiles, students skip count the tiles by 8, hitting a preprogrammed switch or placing a representation of each long foot (1 x 8) in a basket and to determine the total of 64. • Students stop hitting the switch at the end of the length or stop adding a long foot to indicate understanding/performance of area. • Repeat for each style of music. 2. Provide students with a copy of a modified rubric in picture and/or tactile representation and review expectations. 3. Allow students to use all supports provided throughout the unit lesson so far. 4. Students have their own copies of the group results to the dances in the form of visual and/or tactile representations of the dance style paired with the number of people. 5. Students contribute to the class table by handing the paired representations to the teacher.

Lesson 4: Resources

Dance Style	Space Needed for a Person		
	Length	Width	Area
Slow dance	*2 ft*	*2ft*	*$4ft^2$*

Lesson 4: Resources– continued

Dance Style	Space Needed for One Person			Number of People Who Can Dance at One Time
	Length	Width	Area	
Slow dance	*2 ft*	*2ft*	*$4ft^2$*	*16*

Grade Span: 9 - 10	**Content Area:** Math - Geometry
Lesson 5 of the Unit	**Approximate Time Needed:** 90 minutes or two 45 minute blocks

Objectives:

- Identify and quantify attributes of the problem that need to be measured.
- Determine a pattern.
- Generalize relationships.
- Determine the percent of increase/decrease.
- Determine the precision of measurement.

Essential Question(s):

1. What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?
2. How does changing the sides of a square affect the area?
3. How can we use variable expressions to reflect relationships?

Materials Needed:

- Large and small grid graph paper
- Worksheets

Lesson Vocabulary:

Area
Centimeter
Foot
Inch
Length
Meter
Ratio
Unit of Measure
Unit Rate
Width
Yard

Lesson 5: Introduction – 15 minutes

A. Activate Previous Knowledge (unit rate)

1. Review the dance floor problem from the previous lesson with the whole group.

2. Pose a new problem and discuss vocabulary with the class.
   - Alex and Aldo planted several square apple orchards.
   - The table below shows the number of trees and the size of the orchards.

3. Review the information presented in each part of the table. Ask students to analyze the relationship between the side lengths and number of trees. No calculations needed. Keep it general (e.g., in orchard 1, the number of trees to length in feet is 2:1; in orchard 4, the number of trees to feet is 1:1).

Orchard number	Length of each side (feet)	Number of apple trees
1st		
2nd	8 feet	4 trees
3rd	12 feet	9 trees
4th	16 feet	16 trees

Multiple means of representation: Present real life problems using drawings, models, and video representations of orchards of various sizes.

Multiple means of expression: Allow students to present ideas for problem solving using computer models, demonstrations, visuals, etc. Record problem solving ideas in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Students may choose the type of orchard when presenting problem. Allow students to work individually or in small groups based on learning style.

Additional Considerations for Emerging Readers and Emerging Communicators

1. During review, students should refer to their math journals or notes. Be sure students have graphic and/or tactile representations of relevant vocabulary (area, perimeter, length, width) as well as related materials/drawings/object representations from previous lessons.

2. Provide examples through pictures, videos, or tactile representations of orchards of various sizes
   - Point out how much space trees need for maximum growth and production.
   - Ask guiding questions:
     - "Do they have enough space?"
     - "Is there room for more trees?"
   - Students attempt to add more trees to an object representation or virtual representation of the orchard.

B. Establish Goals/Objectives for the Lesson

Inform students that they will make decisions about units and scales that are appropriate for problem solving situations involving mathematics within mathematics or across disciplines or contexts and:

1. Identify and quantify attributes of the problem that need to be measured.
2. Determine a pattern.
3. Generalize relationships.
4. Percent of Increase/ Decrease.
5. Determine the precision of measurement.

Multiple means of representation: Along with posting lesson objectives in the classroom, provide individual copies for students.

Multiple means of expression: Allow students to record lesson objectives in different formats: mathematics journals, computer, premade or original graphic organizers, etc.

Multiple means of engagement: Brainstorm ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. Provide students with keys words paired with symbols/images/ tactile representations (i.e., units of measure, inch, foot, yard, centimeter, and meter).

2. Provide key words in lesson objective paired with symbols/images/tactile representations to record into mathematics journals. Students may use an electronic picture writer to record the lesson objectives.

3. Provide students with photographs, models, or tactile representations of examples of situations in which these concepts are used.

Lesson 5: Body – 30 minutes

Direct Instruction and/or Facilitation of the Lesson

During this portion of the lesson, students will generalize relationships and determine the appropriate scale to express the relationship between two quantities.

1. Review students' ideas on how to solve the orchard problem.
    a. Alex and Aldo planted several square apple orchards.
    b. The table below shows the number of trees and the size of the orchards.

Orchard Number	Length of Each Side (feet)	Area of Each Orchard (ft^2)	Number of Apple Trees
1st	x	?	y
2nd	8 ft	64 ft^2	4 trees
3rd	12 ft	144 ft^2	9 trees
4th	16 ft	256 ft^2	16 trees
5th	x	?	y
n			

2. Students determine the area of each orchard (e.g., The 2nd orchard has an area of 64 ft^2 because 8 x 8 = 64).
3. Given the number of apple trees in each orchard, students determine the square footage needed for each tree using ratios and proportions (e.g., $\frac{area\ of\ orchard}{number\ of\ tree} = \frac{area}{1\ tree}$ or the unit rate of area per tree).
4. Using the ratio from orchard 2, students determine the unit rate (e.g., $\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$ or 64 ft^2 ÷ 4 trees = 16 ft^2 needed for each tree) and confirm that measurement is true for each orchard (i.e., 144 ft^2 ÷ 9 trees = 16 ft^2 and 256 ft^2 ÷ 16 trees = 16 ft^2).
5. Given the measurements in the length of each side column, students determine the rate of change in the length of each orchard (i.e., ___, 8, 12, 16 is a +4 pattern).

Note: Students work in pairs to answer parts 1 - 5 of the problem.

6. Using the rate of change +4, students determine the length of each side (x) for orchards 1 and 5, and fill in the column of the table.

Note: Use whole group discussion for part 6.

7. Students use that information to determine the area of the 1st and 5th orchards (i.e., the 1st orchard has an area of 16 ft^2 because 4ft x 4ft = $16ft^2$ and the 5th orchard has an area of 400 ft^2 because 20ft x 20ft = $400ft^2$).

8. Students use the fact that each tree needs 16 ft^2 to determine how many trees can be planted in the 1st and 5th orchards using $\frac{area\ of\ orchard}{area\ per\ tree}$ = # of trees (i.e., for the 1st orchard, $\frac{16^2}{16^2}$ = 1 tree and for the 5th orchard, $\frac{400^2}{16^2}$ = 25 trees).

9. Students graph the rate of change in the length of each side and the consequent number of trees for each orchard (i.e., (x, y) where x = length of each side and y = the number of apple trees.

Multiple means of representation: Allow students to refer to their brainstorming notes during discussion. When discussing unit rate, provide familiar examples (e.g., miles per hour). Provide students with a copy of the word problem and the table under #1. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem by using formulas and/or models and record information into the tables using various formats: computer, premade or original graphic organizer, etc. Allow students to use a reference of formulas.

Multiple means of engagement: Ensure all students are actively involved in their partnerships. Use scenarios related to students' interests. For example, if a student is interested in animals instead of orchard trees, the scenario could involve the rate of grazing area per horse. Use questioning to encourage students to explain their strategies.

Lesson 5: Body – continued

Additional Considerations for Emerging Readers and Emerging Communicators

1. Refer to the brainstorming ideas for solving the problem. Include picture and/or tactile representations as needed.
    - Provide students with copies of the table as well as word/picture/tactile representations of the words orchard and apple trees.
    - Knowing that the orchards are square, students should determine that the length and width are the same.
    - Students determine the area of orchard #2 by using the formula length x length = area or 8ft x 8ft = 64 ft^2 and/or students may also draw the orchard on grid paper to determine the area.
    - Students can also be provided with a manipulative model or virtual template of orchard #2 so they can determine the area by counting the units.
    - Since the numbers will be quite large, provide students with a means to skip count to determine area.
        - Students can be given units grouped by 8 and a calculator set up to add 8 so each time students place a row of units into the template, they hit enter on the calculator to add 8.
        - Students stop when the template is filled and indicates the final number for area from the calculator.
        - If students are using a computer program, the program would be set up in the same way as lessons 1 and 2.

S**ee Example:** Manipulative worksheets or PowerPoint Lesson 5, Slide 1.

2. Review the concept of unit rate (area/tree) or the amount of space needed per tree.
    - If orchard #2 is 64 ft^2 and has 4 trees, how many square feet is needed for one tree? Students should set up the ratio as $\frac{area\ of\ orchard}{number\ of\ trees}$ or $\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$.
    - Allow students to review strategies used in lesson 3 for using ratios and proportions to solve problems.
    - Students should remember that the equation must remain balanced and that whatever was done to the top portion must be done to the bottom.
    - Since the numbers decrease, students would use division.
      $\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$ or $\frac{64\ ft^2}{4\ trees}$ $\frac{\div 4}{\div 4}$ $\frac{\ ft^2}{tree}$ The unit rate is 16ft^2 per tree.
    - Students use manipulatives to determine how to divide the trees evenly to have a group of one tree (divide by 4).
    - Students should divide the orchard by four as well.

See Example: PowerPoint Lesson 5, Slide 2.

3. Instruct students to determine the rate of change in the unit length of each orchard.
    - Students determine the pattern (x, 8, 12, 16, x; pattern is +4).
    - Students draw each orchard and lay them on top of each other to determine how the side lengths change or add/subtract the difference between unit lengths of the consecutive orchards to determine that each orchard changes by 4 ft.
    - Students use that information to determine the unit length of orchard #1 and orchard #5.
        - For example: length of orchard #1 +4ft = length of orchard #22

          $x+4ft = 8ft$

          $x + 4ft - 4ft = 8ft - 4ft$

          $x=4ft$
        - For example: length of orchard #4 + 4 ft = length of orchard #5

          $16\ ft + 4\ ft = x$

          $20\ ft = x$

See Example: PowerPoint lesson 5, Slide 3.

4. Students use the information of unit length to determine the area of orchards #1 and #5.
    - Students use given formula for a square (length x length = area) and/or draw the orchards on grid paper and count the squares to determine the area.
    - Students use manipulatives of the rate of change by placing a unit length of four, starting at the end of the length and width on orchard #2 to determine the unit length for orchard #1.
    - Students draw or model the orchards on grid paper and count the squares to determine the area.

See Example: PowerPoint Lesson 5, Slide 4.

    - Students use the rate of change by adding it onto the length of orchard #4 to determine the length of orchard #6.

See Example: PowerPoint Lesson 5, Slide 5.

5. Now that students have determined the unit rate, they determine the number of trees per orchard for orchard #5.
    - Using unit length determined in step 3, students use the formulas and ratios to determine the area of the orchard and the number of trees that can be planted in the orchard.
    - Students can also use grid paper to draw orchard #5 (based on the dimensions of 20 ft x 20 ft) and by using a cut out of the unit rate, determine how many trees can be planted.

See Example: Lesson 5 or students use virtual manipulatives as in PowerPoint Lesson 5 Slide 2.

6. Tell students they will be graphing the relationship between the size of the garden and the number of trees that can be planted.
    - Provide students with a coordinate grid with the x- and y- axes labeled.
    - Students must use the information from their tables to create ordered pairs and complete the graph.
    - The columns from which the ordered pairs are created are already labeled as x and y, but they can also be highlighted as needed.
    - Students count over (the run) for x and up (the rise) for y, or students find the matching number for x and move the point up to the matching number for y.

See Example: PowerPoint Lesson 5, Slide 6.

Important Consideration: For some students, the difficulty/complexity can be reduced by using only the first quadrant of the coordinate grid.

Lesson 5: Practice – 30 minutes

1.Provide similar problems and additional practice questions based on students' responses.

- For example, Casey and Liz want to plant their own square apple orchard.
- They decide to increase the sides of Alex and Aldo's 3rd orchard by 25%.
- If they keep the same area per tree, how many trees can they plant in their square orchard?

Multiple means of representation: Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem by using formulas and/or models and record information into the tables using various formats: computer, premade or original graphic organizer, etc. Allow students to use a reference of formulas.

Multiple means of engagement: Ensure all students are actively involved in their partnerships. Use scenarios related to students' interests. For example, if a student is interested in animals instead of orchard trees, the scenario could involve a rate of grazing area per horse. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1.Provide students with copies of the problem paired with picture and/or tactile representations. Ensure students have the table used during instruction to refer to for solving this problem.

2. Students determine the area of Casey's and Liz's orchard by multiplying the area of orchard #3 by 1.25.

3. Students may also use a drawing of orchard #3 on grid paper and divide it evenly in fourths (or quarters) to determine what 25% more would be and combine the quarter representation with orchard #3.

See Example: Manipulatives or PowerPoint Lesson 5, Slides 7 & 8.

4. Once students have determined the area of Casey and Liz's orchard, they can determine how many trees can be planted based on the unit rate of $\frac{16\ ft^2}{1\ tree}$.

5. Students should use the same strategies and supports as they used previously.

Lesson 5: Closure - 15 minutes

a. Revisit/Review Lesson and Objectives

Remind students that they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts and:

1. Identify and quantify attributes of the problem that need to be measured.
2. Determine a pattern.
3. Generalize relationships.
4. Percent of increase/ decrease.
5. Determine the precision of measurement.

Multiple means of representation: Along with posting lesson objectives in the classroom, students may refer to their individual copies.

Multiple means of expression: Students can share what they have learned in different formats: writing, drawing, creative expression, etc.

Multiple means of engagement: Share ideas of how and when these skills might be relevant to "me."

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.

2. Students use the information recorded in their journals to refer back to the lesson objectives' key words paired with images. From that information, they share what they have learned based on each of the expectations.

a. For example, students may grab the tactile cue for area to state "I have learned that the area is all the space measured within a figure."

3. Students should also refer back to the photographs of examples of real-life situations when these concepts are used to share when they could use these new skills.
    - For example, students could touch the tactile representations for area and orchard to state, "I can use the unit rate of area to tree to determine the size of the orchard."

B. Exit Assessment

1. Students work either in pairs or individually to produce their own word problems similar to the ones presented in this lesson. Once the problems are written, students identify the unit rate of their problem (e.g., area per tree or area per person). If time permits, students can trade problems and solve them as a review for another in-class activity.

Multiple means of representation: Ensure students have the previous word problems from this lesson and/or lesson 4 to review and model. Have previous drawings, models, and manipulatives available for students to use.

Multiple means of expression: Allow students to create the problem using various formats: computer, premade or original graphic organizer, models, etc. Allow students to use a reference of formulas.

Multiple means of engagement: Ensure all students are actively involved in creating their problems. Encourage students to use scenarios related to their interests. For example, if a student is interested in animals instead of orchard trees, the scenario could involve a rate of grazing area per horse. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers and Emerging Communicators

1. Students should have access to a variety of picture representations and models as they brainstorm ideas for their problems. Provide choices of interest to students in picture/tactile format.

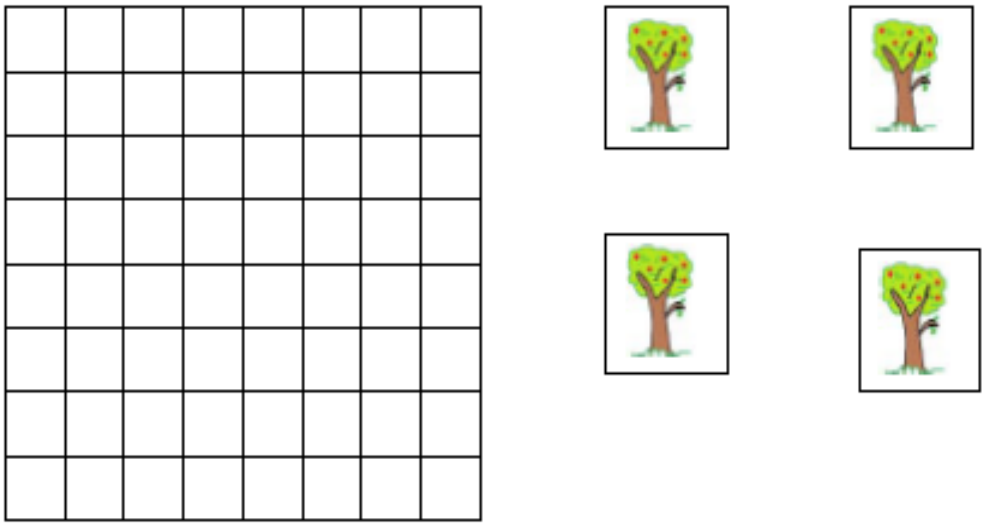
2. Students review the information in previous problems and choose the key words paired with images to use when creating their problems or provide students with a template of a word problem that they can complete with key words and unit rate concepts.

Lesson 5: Resources

Alex and Aldo planted several square apple orchards. The table below shows the number of trees and the size of the orchards (See ppt demonstration).

Orchard Number	Length of Each Side (feet)	Area of Each Orchard (ft^2)	Number of Apple Trees
1st	x	?	y
2nd	8 ft	64 ft^2	4 trees
3rd	12 ft	144 ft^2	9 trees
4th	16 ft	256 ft^2	16 trees
5th	x	?	y
n			

Orchard #2 is 8ft x 8ft = $64ft^2$ and has 4 trees.

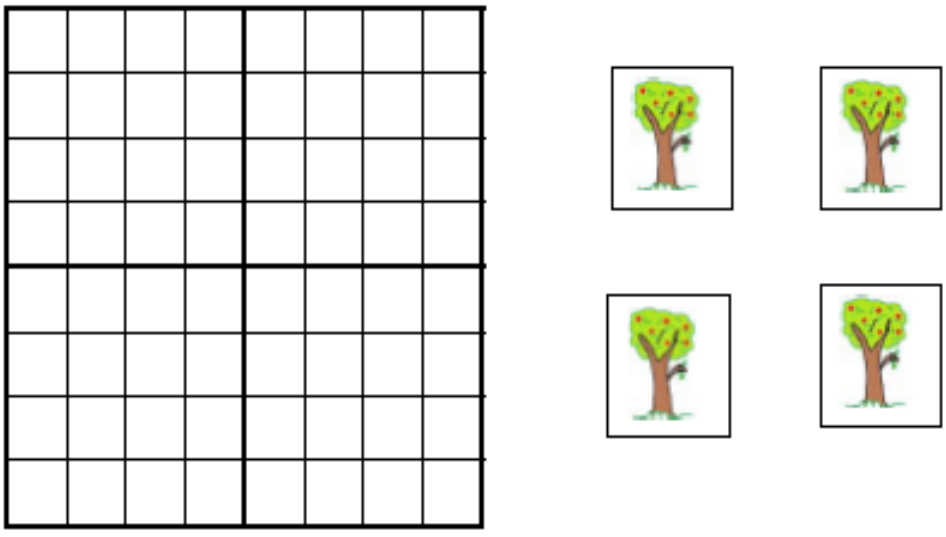


How much space is needed for one tree?

$\frac{64^2}{4\ trees} = \frac{?^2}{1\ tree}$ to get one tree, student should divide the group of trees by four $\frac{64^2}{4\ trees \div 4} = \frac{?^2}{1\ tree}$.

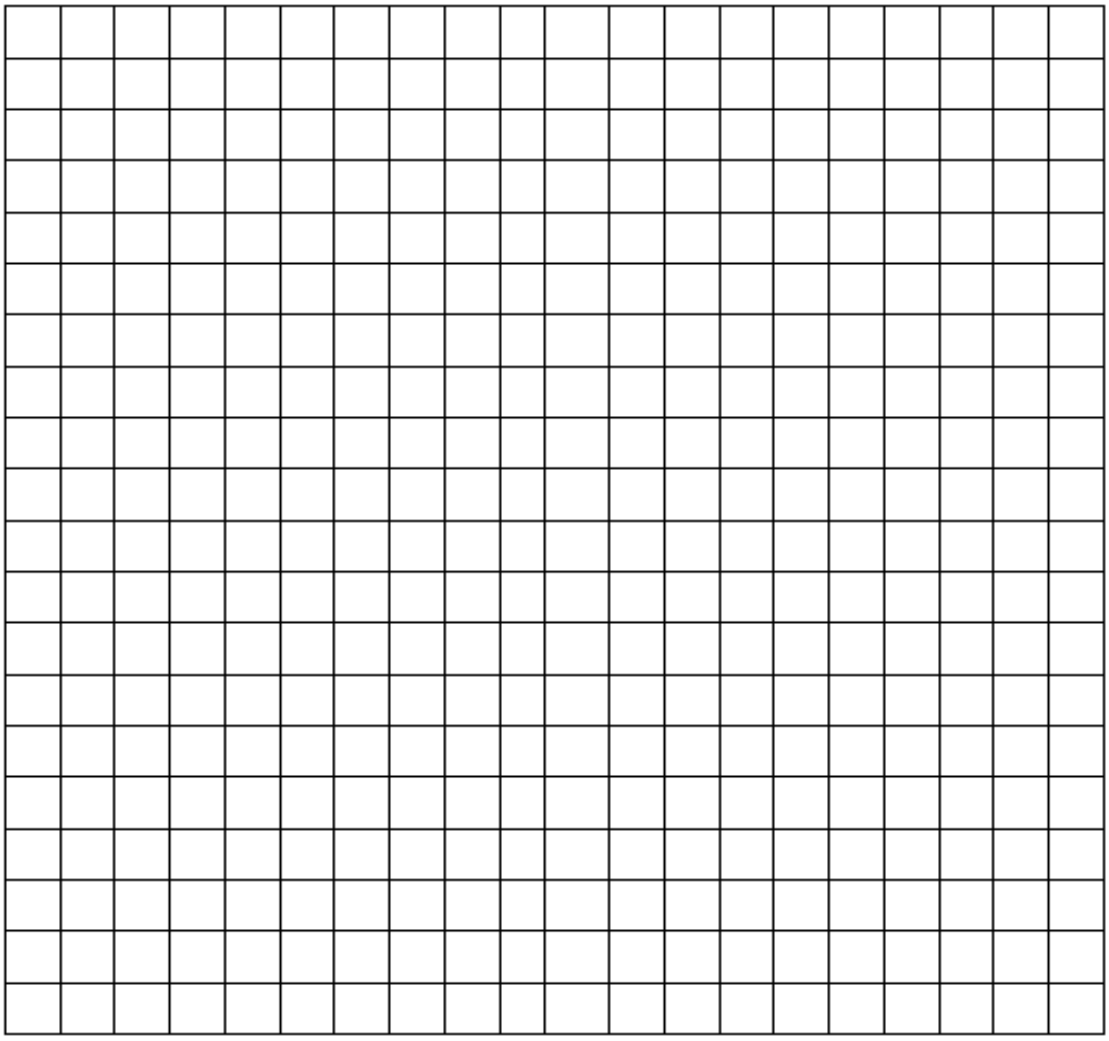
If the student divides the trees by four, she/he must divide the area of the orchard by four

$\frac{64^2 \div 4}{4\ trees \div 4} = \frac{?^2}{1\ tree}$

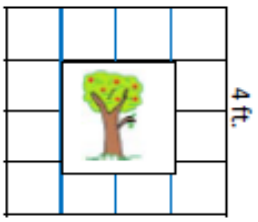


Orchard #5 is 20ft x 20ft

20 ft.

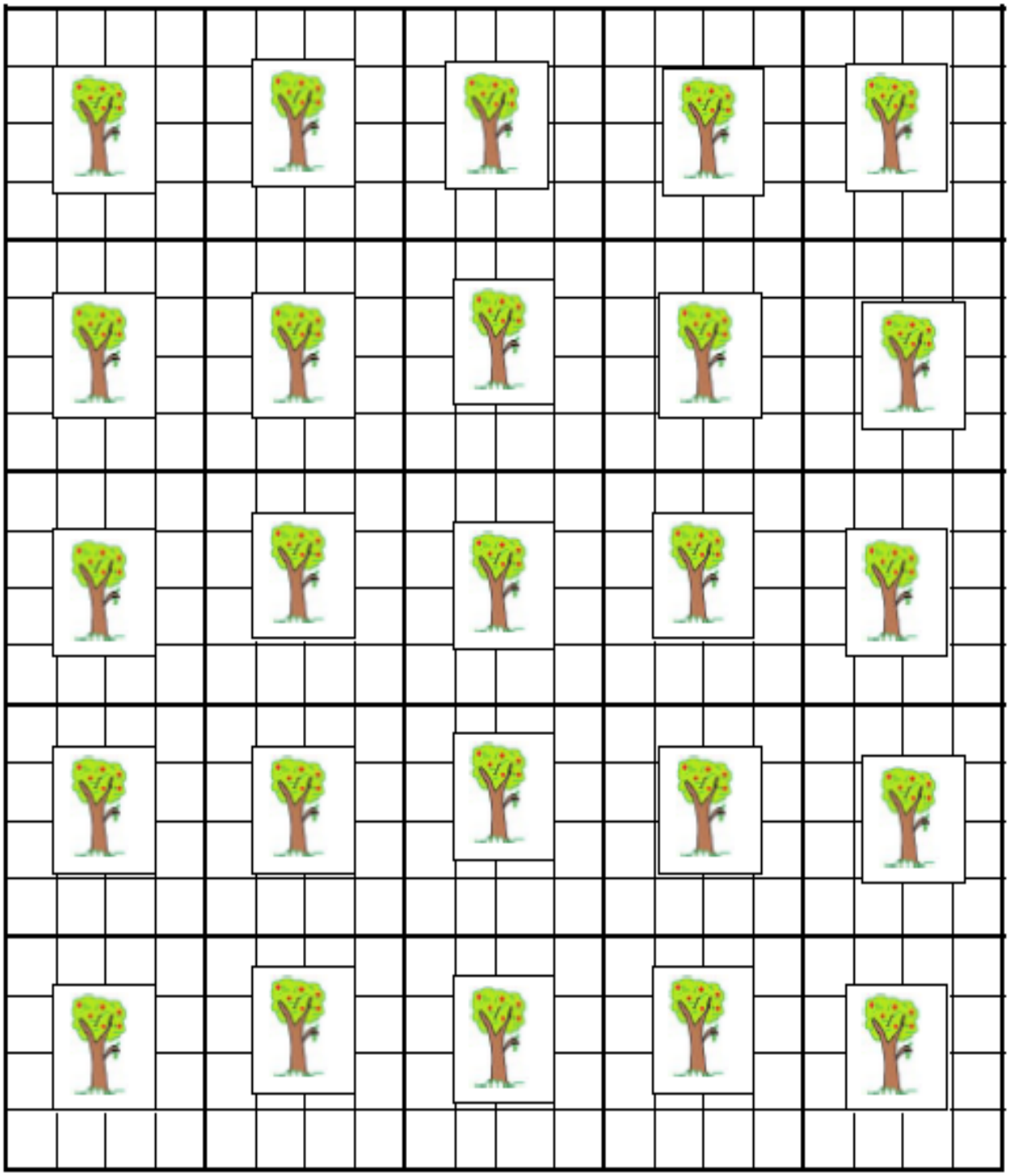


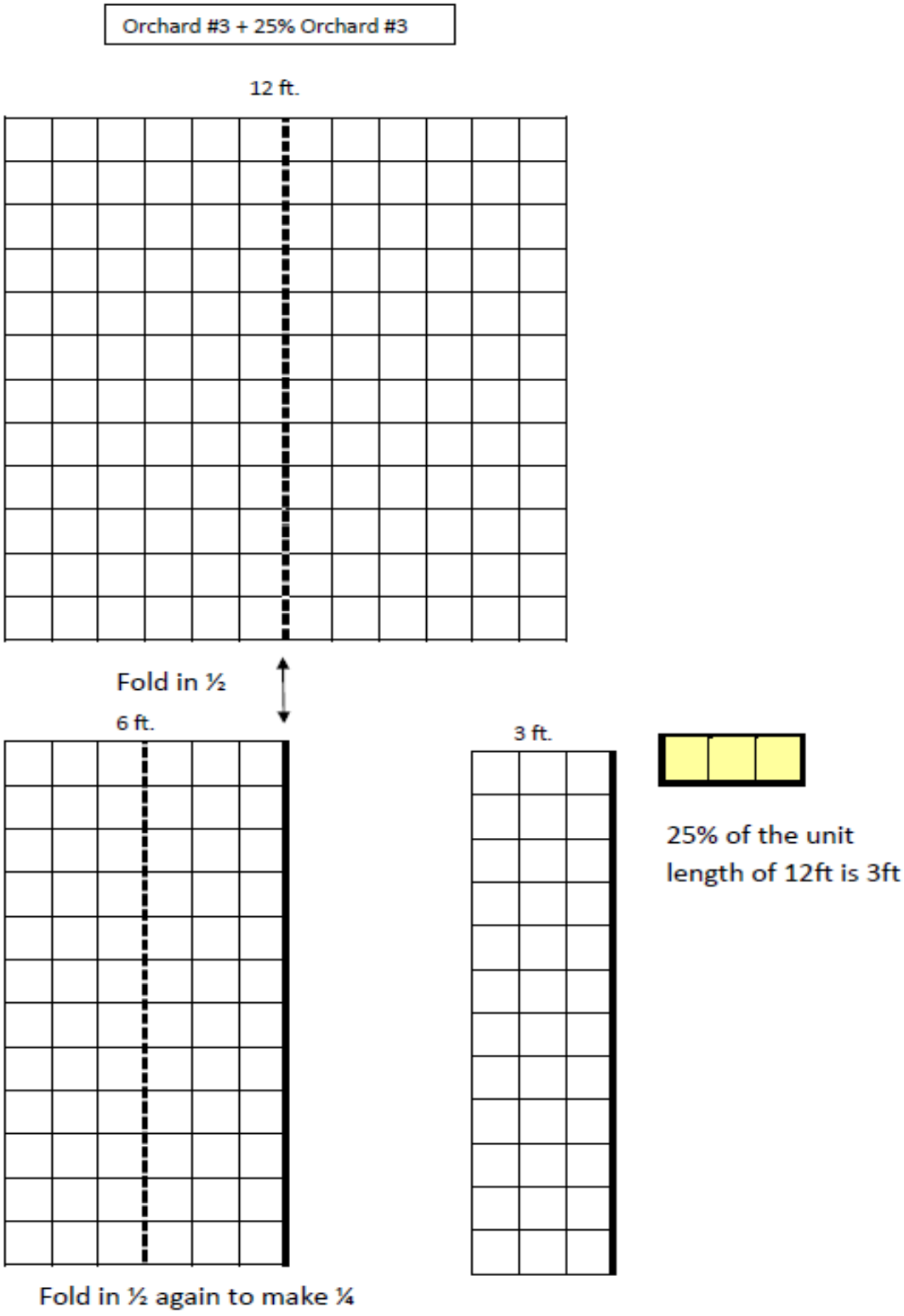
Unit rate $16ft^2$ per tree



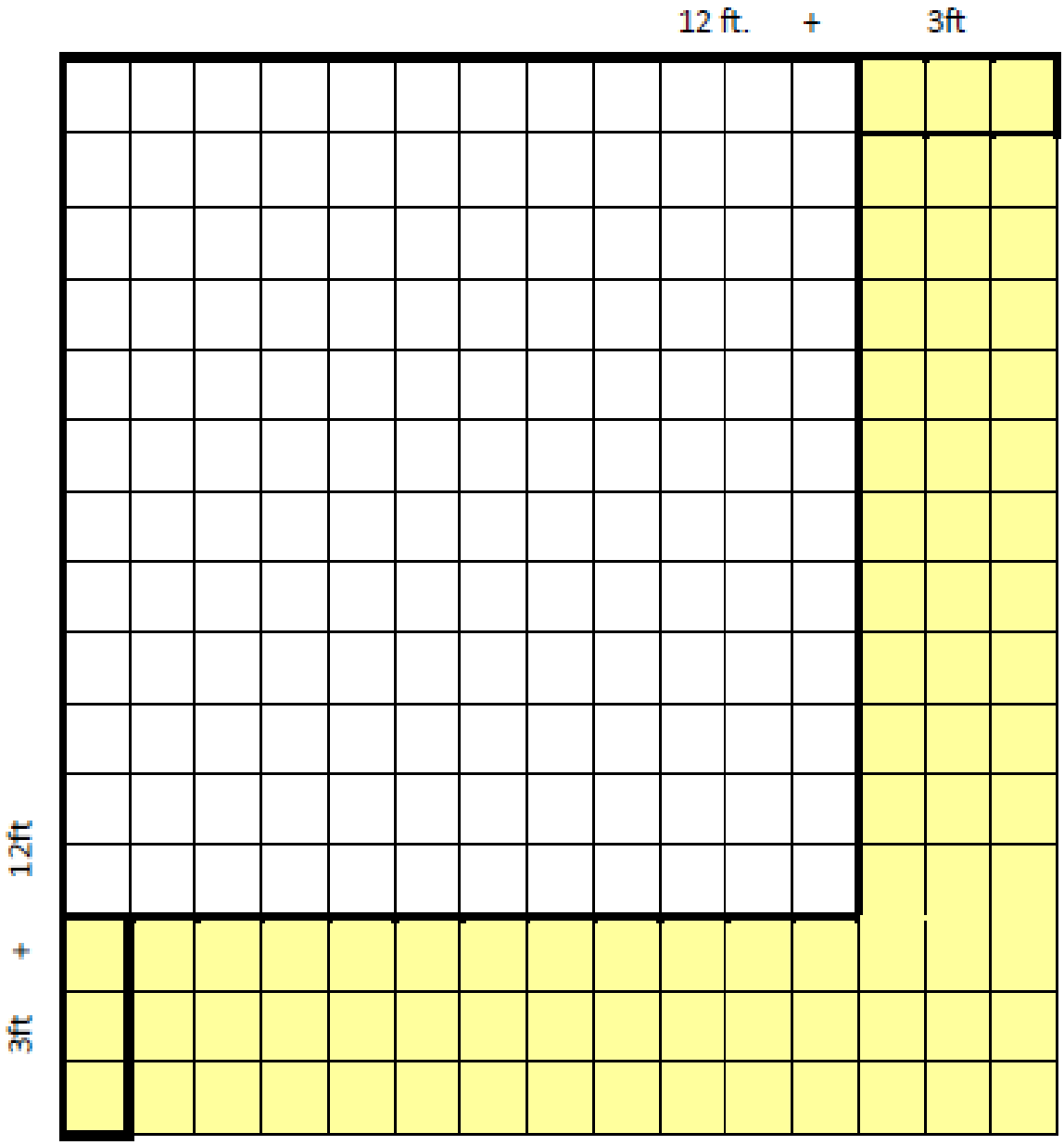
The unit rate is area per tree:

20 ft.





Orchard #3 + 25% Orchard #3



Grade Span: 9 - 10	**Content Area:** Investigating Measurement in the Real World – *Culminating Activity*
Lesson 6 of the Unit	**Approximate Time Needed:** 45 minutes

Objectives:

- Identify and quantify attributes of the problem that need to be measured.
- Determine a pattern.
- Generalize relationships.
- Determine percent of increase/decrease.
- Determine the precision of measurement.

Essential Questions:

1. What are the relationships among the measurements of dimensions, area, and perimeter in problem solving situations?
2. How can we use variable expressions to reflect relationships?
3. How do we determine limitations of measurement

Materials Needed:

- Large and small grid graph paper
- Worksheets

Lesson Vocabulary

Area
Centimeter
Conversion
Foot
Inch
Length
Meter
Perimeter
Proportion
Ratio
Rectangles
Similar Rectangles
Unit of Measure
Unit Rate
Width
Yards

Lesson 6: Culminating Activity - 45 minutes

CULMINATING ACTIVITY FOR THE UNIT.

A. Revisit/Review Unit and Lesson Objectives

Remind students that throughout these lessons they were to make decisions about units and scales that are appropriate for problem solving situations within mathematics or across disciplines or contexts, and:

1. Convert units using standard/known conversion units.
2. Use appropriate known formulas for the area.
3. Solve multistep problems involving one unit of measure.
4. Set up and solve proportions.
5. Convert units of measurement using standard/known conversions.
6. Recognize when to multiply and when to divide in converting measurements.
7. Use ratio and proportion to convert measurements.
8. Use appropriate known formulas for area.
9. Identify, quantify, and compare the attributes of the objects, situations, and/or events that need to be measured to solve the problem/situation.
10. Use appropriate units of measure to identify, quantify, and compare objects, situations, and/or events to solve a real world problem.
11. Convert units when necessary.

Conduct a class discussion on which skills were used to solve different types of problems. Discuss the additional strategies they used to implement the skills and solve the problems.

Multiple means of representation: Along with posted lesson objectives in the classroom, students may refer to their individual copies of the objectives and their mathematics journals.

Multiple means of expression: Students share what they have learned or strategies they have used by showing different models, pictures, drawings, etc. used throughout the lessons.

Multiple means of engagement: Share ideas of how these skills have been useful in solving the problems from previous lessons and what strategies were the most helpful.

Additional Considerations for Emerging Readers and Emerging Communicators

1. When reviewing the expected outcomes, have students refer to the lesson objectives they recorded in their mathematics journals or their electronic picture versions.
2. Students use the information recorded in their journals to refer back to the lesson objectives' key words paired with images. From that information, they share what they have learned based on each of the expectations.
   - For example, the student may grab the tactile cue for area to state, "I have learned that the area is all the space measured within a figure."

3. Students refer to examples of their work to demonstrate how the skills and strategies were used.
    - For example, a student could touch the tactile representations for area and orchard to state, "I used the concept of area to determine how much space a tree needs to grow."

B. Exit Assessment

Scenario:

The freshman class officers at Riverside High School are planning the annual Freshman Class Winter Dance. They have decided to hold the dance at the White Oak Country Club. The room they have reserved for the dance is carpeted, but for events such as this dance, the country club places parquet flooring over the carpet to make a dance floor.

The parquet flooring is laid in interlocking sections, and so the dance floor can be arranged to be various sizes to accommodate the number of dancers attending the event. However, the class officers need to let the country club managers know a month in advance of the dance how big to make the dance floor so the club workers will have enough time to get the flooring laid out appropriately. The class officers decide to base that decision on their latest ticket sales.

Things to Consider:

- The minimum size of the dance floor being considered by the class officers is 30 ft x 30 ft.
- The class officers assume 30 ft x 30 ft will be enough room for 100 couples to be on the dance floor at once.
- The country club workers can increase the sides of the dance floor in 5 ft increments in either direction, but the class officers want to maintain a square dance floor.
- The maximum the 30 ft sides of the dance floor can be increased is by 50%.

Tasks:

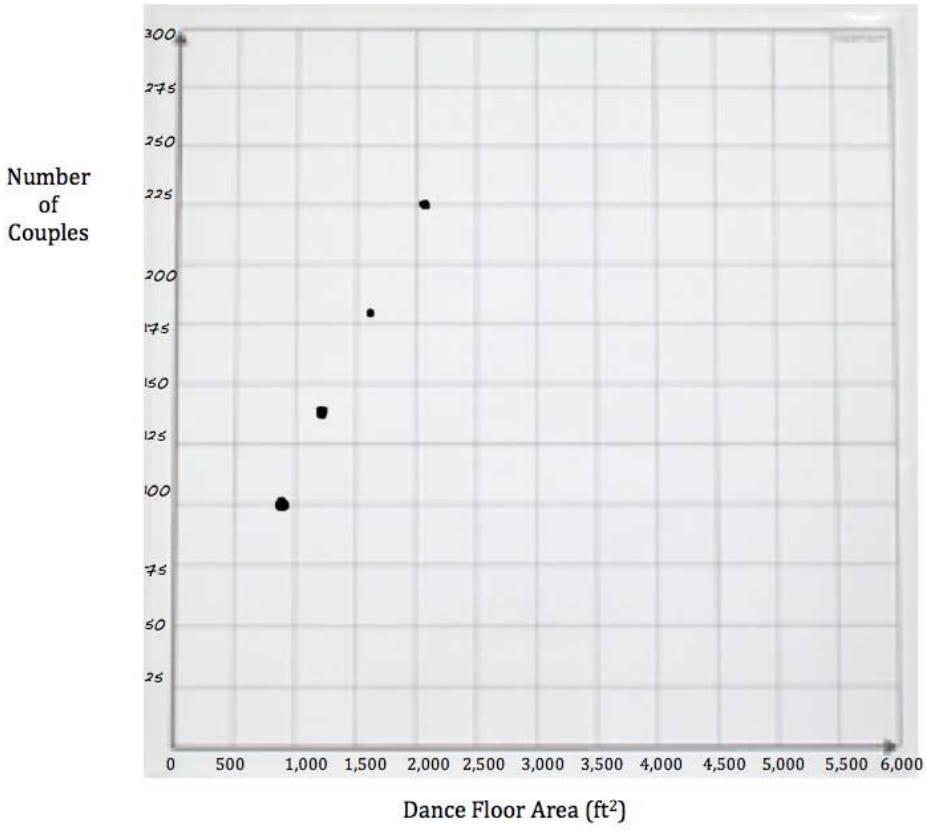
1. Determine how many square feet of dancing space each couple would have if the dance floor is 30 ft x 30 ft (i.e., 30 ft x 30 ft = 900 ft^2 and 900 ft^2 ÷ 100 couples = 9 ft^2 for each couple).
2. Determine the maximum length of the sides of the dance floor (i.e., 30 ft x 50% = 15 ft and 30 ft + 15 ft = 45 ft as the maximum length for the dance floor).
3. Using *n* to represent the number of possible increases, solve for *n* having determined that the 50% increase to 30 ft would be an additional 15 ft and that the increments are made 5 ft at a time (i.e., n = 15 ft ÷ 5 ft increments = 3 increases [35, 40, and 45]).
4. Determine the area of the dance floor for each of the 3 increases, and determine the number of couples that each would accommodate (i.e., 45 ft x 45 ft = 2,025 ft^2 and 2,025 ft^2 ÷ 9 ft^2 needed per couple = enough space for 225 couples).

5. Make a chart showing all of the dance floor size possibilities as well as the number of couples that could be accommodated by each.

	Side Lengths	Area of the Dance Floor	Number of Couples
	30 ft x 30 ft	900 ft^2	100 couples
1st increase	35 ft x 35 ft	1,225 ft^2	136 couples*
2nd increase	40 ft x 40 ft	1,600 ft^2	177 couples*
maximum increase	45 ft x 45 ft	2,025 ft^2	225 couples

* 1,225 ft^2 ÷ 9 ft^2 actually = 136.1 and 1,600 ft^2 ÷ 9 ft^2 actually = 177.777778 so, there would be a little space left over but not enough for another couple.

6. Create a graph that shows the relationship between the increase in the size of the dance floor and the number of couples who can attend the dance. Make sure to scale and label your axes (i.e., (x,y) where x = dance floor area and y = number of couples).



7. Given that 165 couples' tickets have been sold, suggest how large the class officers should tell the country club managers the floor will need to be and explain why it should be 40' x 40' because that area can accommodate up to 177 couples whereas the 35' x 35' could only accommodate 136 couples.

Multiple means of representation: Allow students to refer to their mathematics journals and other notes as they solve the problem. Provide students with a copy of the word problem and the table. Have drawings and manipulatives available for students to use.

Multiple means of expression: Allow students to solve the problem using formulas and/or models and record information into the tables using various formats: computer, paper pencil, drawings, etc.

Multiple means of engagement: Ensure all students are actively involved in solving the problem. Encourage students to consider options for solving the problem that will engage them. Use questioning to encourage students to explain their strategies.

Additional Considerations for Emerging Readers

1. Provide the written problem to students paired with picture symbols and/or tactile cues as well as the things to consider list.
2. Allow students to use a text reader to initially access the problem and to go back and review the problem as needed.
3. Provide the written questions (1-7) to students paired with picture symbols and/or tactile cues.
4. Provide students with a variety of formulas, including the ratio for the unit rate ($\frac{area\ of\ floor}{\#\ of\ couples}$), area, and perimeter as well as a copy of the table below.
5. Students should have grid paper to create a model of the area of the dance floor as well as manipulatives representing both variables (area of dance floor and number of couples).
6. Students first need to determine the area of the dance floor by choosing and using the formula for area.
7. Students need to determine the unit rate.

8. Students use the formula chosen from options ($\frac{area\ of\ floor}{\#\ of\ couples} = \frac{area\ of\ floor}{1\ couple}$; $\frac{900\ ft^2}{100} = \frac{area\ of\ floor}{1}$), or manipulatives, etc.

	Side Lengths	Area of the Dance Floor	Number of Couples	Unit Rate area/couple
	30 ft x 30 ft	(Question #1)	100 couples	
1st increase				
2nd increase				
3rd increase				

9. Determine the maximum length of the sides of the dance floor (i.e., 30 ft x 50% = 15 ft and 30 ft + 15 ft = 45 ft as the maximum length for the dance floor).
10. Be sure students have the following "things to consider" in picture format and or electronic text reader to refer to when answering question 2:
    - The country club workers can increase the sides of the dance floor in 5 ft increments in either direction, but the class officers want to maintain a square dance floor.
        - Students create a unit length representing 5 ft or are given choices of various unit lengths, one of which is 5 ft that can be used to extend the dance floor.
        - Students could also have a template that represents + 5 to use to complete the column in the chart on floor lengths.
    - The maximum the 30 ft sides of the dance floor can be increased is by 50%.
11. Provide students with a variety of strategies for determining half of 30 (divide by two, fold representation of 30 ft x 30 ft dance floor in half and count half side length, etc.)
12. Using *n* to represent the number of possible increases, solve for *n* having determined that the 50% increase to 30 ft would be an additional 15 ft and that the increments are made 5 ft at a time (i.e., n = 15 ft ÷ 5 ft increments = 3 increases [35, 40, and 45]).
13. Students may use a drawing of the 30 ft x 30 ft dance floor on grid paper and divide it evenly in half to determine what 50% more would be.
14. Students use the 5 ft unit length template to determine how many increments of 5 the floor can be increased and what the measurement would be for each increase (i.e. 30 ft + 5 ft = 35 ft; 35 ft + 5 ft = 40 ft, etc.). Students may also determine the increase by counting by 5s from 30.
15. Determine the area of the dance floor for each of the 3 increases and determine the number of couples that each would accommodate (i.e., 45 ft x 45 ft = 2,025 ft^2 and 2,025 ft^2 ÷ 9 ft^2 needed per couple = enough space for 225 couples).

a. Students must remember to increase both the length and width by 5 ft and use the formula and/or manipulatives to determine the area of each floor increase.

16. Using the unit rate determined in step 1, students choose the correct given formula to use to determine how many couples can be accommodated for each increase, and/or students use manipulatives of the different floor measurements and the unit rate to determine how many couples can dance per floor size.
17. Make a chart showing all of the dance floor size possibilities as well as the number of couples that could be accommodated by each.
18. Students can be given the above chart to complete from results found in steps 1, 2, and 3.
19. Create a graph that shows the relationship between the increase in the side length of the dance floor and the number of couples that can attend the dance.
    - Students should scale and label the coordinate grid (first quadrant), using the side length of each dance floor, to keep the numbers manageable. (i.e., (x,y) where x = side length of the dance floor and y = number of couples; where the scale of x increases by 5's to 50 and the scale of y increases by 25 to 250).
    - Students can be given choices of scaled and labeled coordinate grids and choose the best representation for the problem and then plot the points on the chosen grid.
    - Students should use the table to determine the ordered pairs needed to plot the points.
    - Students identify and highlight the two variables used for graphing (side length and number of couples).
20. Finally, given that 165 couples' tickets have been sold, suggest how large the class officers should tell the country club managers the floor will need to be and explain why the dance floor should be 40' x 40' because that area can accommodate up to 177 couples whereas the 35' x 35' could only accommodate 136 couples.
    - Students should use their table and graph to determine the appropriate floor size.
    - Students explain using cloze sentences (e.g., The dance floor should be _____ because that area can accommodate up to _____ couples whereas the _____ could only accommodate _____ couples.)

Additional Considerations for Emerging Communicators

. Provide the written problem to students paired with picture symbols and/or tactile cues.

- Allow students to use a text reader to initially access the problem and to go back and review the problem as needed.
- Provide models of the initial problem as well as for the things to consider list.

- Modify the things to consider section to use smaller numbers for the side lengths and number of couples to start the problem.

Things to Consider:

- The minimum size of the dance floor being considered by the class officers is 12 ft x 12 ft.
- The class officers assume 12 ft x 12 ft will be enough room for 16 couples to be on the dance floor at once.
- The country club workers can increase the sides of the dance floor in 2 ft increments in either direction, but the class officers want to maintain a square dance floor.
- The maximum the 12 ft sides of the dance floor can be increased by is 50%.

Tasks:

1. Provide students with a variety of formulas, including the ratio for the unit rate ($\frac{area\ of\ floor}{\#\ of\ couples}$), area, and perimeter as well as a copy of the table.
2. Students should have grid paper to create a model of the area of the dance floor in question one as well as manipulatives representing both variables (area of dance floor and number of couples).
3. Students first need to determine the area of the dance floor by choosing and using the formula for area.

	Side Lengths (x)	**Area of the Dance Floor**	**Number of Couples (y)**	**Unit Rate (Area/couple)**
	12 ft x 12 ft	(Question #1)	16 couples	
1st increase				
2nd increase				
3rd increase				

4. Determine the maximum length of the sides of the dance floor (i.e., 12 ft x 50% = 6 ft and 12 ft + 6 ft = 18 ft as the maximum length for the dance floor).

5. Be sure students have the following things to consider in picture format and/or electronic text reader to refer to when answering question 2:
    - The country club workers can increase the sides of the dance floor in 2 ft increments in either direction, but the class officers want to maintain a square dance floor.
    - Students can be given choices of various unit lengths, one of which is 2 ft that can be used to extend the dance floor.
    - The maximum the 12 ft sides of the dance floor can be increased by is 50%.
    - Provide students with a variety of strategies for determining half of 12 (divide by two, fold representation of 12 ft x 12 ft dance floor in half and count half side length, etc.).
6. Using *n* to represent the number of possible increases, solve for *n* having determined that the 50% increase to 12 ft would be an additional 6 ft and that the increments are made 2 ft at a time (i.e., n = 6 ft ÷ 2 ft increments = 3 increases [14, 16, and 18]).
7. Determine the area of the dance floor for each of the 3 increases, and determine the number of couples that each would accommodate (i.e., 18 ft x 18 ft = 324 ft^2 and 324 ft^2 ÷ 9 ft^2 needed per couple = enough space for 36 couples).
8. Students must remember to increase both the length and width by 2 ft and use manipulatives and/or computer with virtual manipulatives to determine the area of each floor increase.
9. Using the unit rate determined in step 1, students should us the manipulative/template of the different floor measurements and the unit rate to determine how many couples can dance per floor size.
10. Make a chart showing all of the dance floor size possibilities as well as the number of couples that could be accommodated by each.
11. Students can be given the above chart to complete from results found in steps 1, 2, and 3
12. Students should refer to the representations used/created in each step above and multiple choice options to complete the table.
13. Create a graph that shows the relationship between the increase in the size of the dance floor and the number of couples that can attend the dance (make sure to scale and label axes) [i.e., (x,y) where x = dance floor area and y = number of couples).
14. The graph should be scaled and labeled using the side length of each dance floor, to keep the numbers manageable. [i.e., (x,y) where x = side length of the dance floor and y = number of couples; where the scale of x increases by 2's to 20 and the scale of y increases by 2 or 4 to 40).
15. Students can be given choices of scaled and labeled coordinate grids and choose the best representation for the problem and plot the points on the chosen grid.

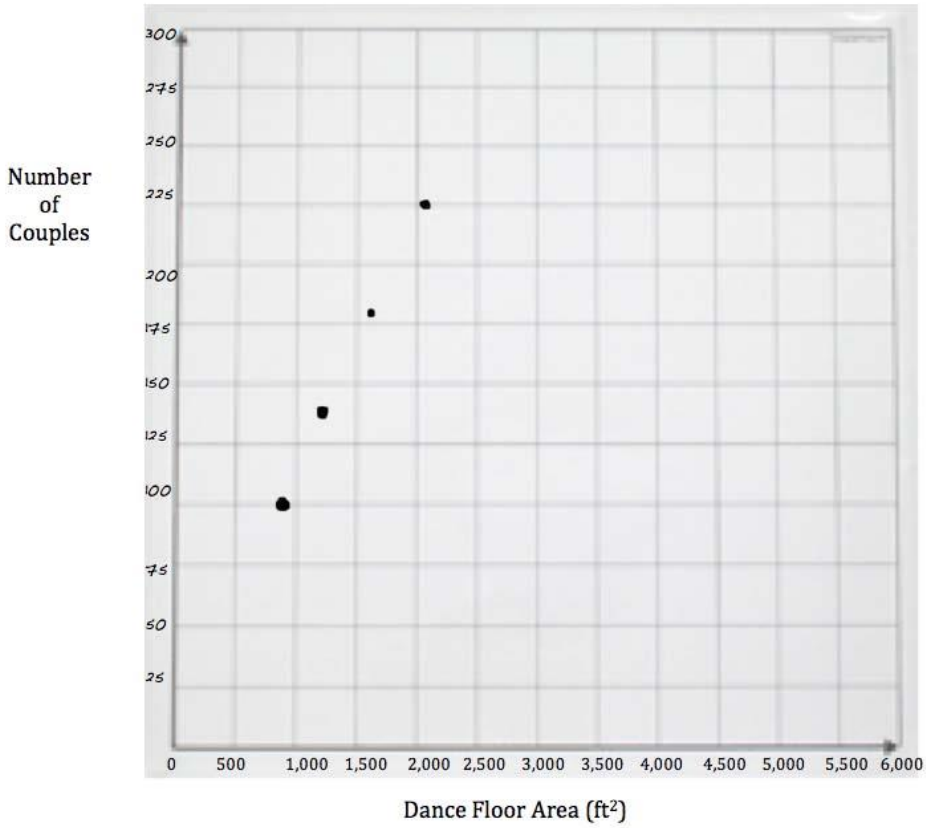
16. Students should use their completed table to determine the ordered pairs needed to plot the points.
17. The x and y variables can be highlighted for graphing (side length and number of couples).
18. Students identify where the points should go by first identifying the number representing the independent variable (x) and the correct corresponding number representing the dependent variable (y), or students use a computer program to graph by indicating the numbers in an order pair.

See Example: PowerPoint Lesson 5, slide 6

19. Finally, given that 32 couples' tickets have been sold, suggest how large the class officers should tell the country club managers the floor will need to be and explain why it should be 18' x 18' because that area can accommodate up to 36 couples whereas the 16' x 16' could only accommodate 28 couples.
20. Students should use their table and graph to determine the appropriate floor size.
21. Students indicate which floor size by comparing the number of couples attending to the number of couples that can fit on the dance floor and choosing the floor with immediately higher/larger number.
22. Students explain using cloze sentences (e.g., The dance floor should be _____ because that area can accommodate up to _____ couples whereas the _____ could only accommodate _____ couples.)

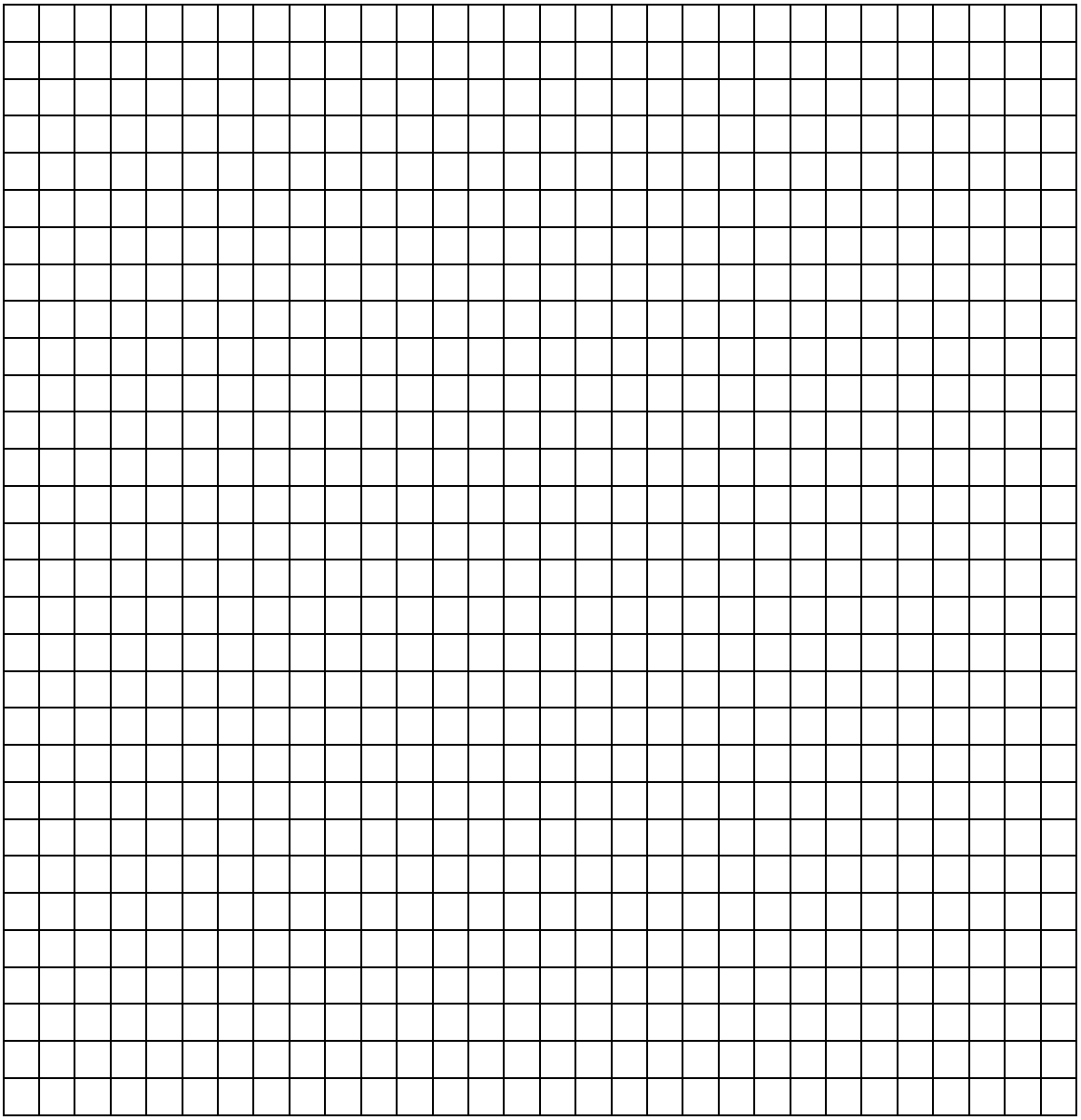
Lesson 6: Resources

	Side Lengths	Area of the Dance Floor	Number of Couples
	30 ft x 30 ft	900 ft^2	100 couples
1st increase	35 ft x 35 ft	1,225 ft^2	136 couples*
2nd increase	40 ft x 40 ft	1,600 ft^2	177 couples*
maximum increase	45 ft x 45 ft	2,025 ft^2	225 couples



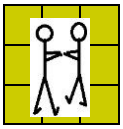
Lesson 6: Resources– continued

30 ft



30 ft

Unit rate 9 ft^2 per couple



Lesson 6: Resources– continued

	Side Lengths	Area of the Dance Floor	Number of Couples	Unit Rate area/couple
	30 ft x 30 ft	(Question #1)	100 couples	
1st increase				
2nd increase				
3rd increase				

Lesson 6: Resources– continued

	Side Lengths (x)	Area of the Dance Floor	Number of Couples (y)	Unit Rate (Area/couple)
	12 ft x 12 ft	(Question #1)	16 couples	
1st increase				
2nd increase				
3rd increase				

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